Review 1 Spring 1989

1. Simplify

   a) $\ln e^{\sqrt{5}}$ b) $\sqrt{e^{\ln 4}}$ c) $e^{2\ln 3 - 3\ln 2}$

   d) $\cos\left(\sin^{-1}\dfrac{5}{12}\right)$ e) $\tan^{-1}\left(\tan\dfrac{3\pi}{4}\right)$

2. State the definition of $\sin^{-1} x$.

3. Find $\frac{dy}{dx}$

   a) $y = (\ln x)^{-3}$ b) $y = x^{\pi} + \pi^{x}$ c) $y = \ln(e^{2x} + 1)$

   d) $y = (x^2 + 1)^{2x}$ e) $y = \ln\left[\dfrac{(5x+1)^3(x-1)^{\frac{2}{3}}}{\sqrt{x^2+4}}\right]$

4. Find

   a) $\displaystyle\int \frac{x^2}{1-x^3}\,dx$ b) $\displaystyle\int_e^{e^2} \frac{dx}{x(\ln x)^2}$ c) $\displaystyle\int 5^x\,dx$

   d) $\displaystyle\int xe^{(1+x^2)}\,dx$ e) $\displaystyle\int \frac{1+e^{3x}}{e^{3x}}\,dx$

5. Find the solution to the differential equation

$$x^2\frac{dy}{dx} = y^2$$

   that satisfies $y(1) = 2$.

   Write your answer as a formula for y in terms of x.

6. In the decay of a radioactive substance, the rate of change of the mass y of the substance is directly proportional to y. Suppose that after 10 years the original 1000 grams of the substance have been reduced to 500 grams. How many grams will remain after 15 years?

7. Let $f(x) = \dfrac{\ln x}{x^2}$. Compute $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to 0^+} f(x)$.

   Find all critical points and inflect points for f. Sketch the graph of f.