Math 12 | Department of Mathematics | Fall 1996

Exam 1 Review

1. Find the value

(a) $e^{3\ln 2}$ (b) $\ln e^{-0.3}$ (c) $\log_{10}\left(\frac{1}{100}\right)$ (d) $\log_{100}\left(\frac{1}{10}\right)$

(e) $\sin^{-1}\left(\sin\left(-\frac{\pi}{5}\right)\right)$ (f) $\cos^{-1}\left(\cos\left(-\frac{\pi}{5}\right)\right)$ (g) $\sin\left(\sin^{-1}(-.6)\right)$ (h) $\cos\left(\sin^{-1}(-.6)\right)$

2. Find $\frac{dy}{dx}$

(a) $y = e^{3x^2-7}$ (b) $y = \ln(7x^3+2)$ (c) $y = \tan^{-1}(2x+5)$ (d) $y = \pi^x + x^\pi$

(e) $y = \log_2 x$ (f) $y = \frac{(x^2+7)^3 e^{7x}}{x^{4/3}(3x+1)}$ (g) $y = (x^2+1)^{2x+7}$ (h) $y = \sin^{-1}(e^{2x})$

3. Evaluate

(a) $\int xe^{3x}\,dx$ (b) $\int xe^{3x^2}\,dx$ (c) $\int e^{2x}\sin 3x\,dx$ (d) $\int \frac{1}{4+x^2}dx$

(e) $\int x\ln x\,dx$ (f) $\int \frac{\ln x}{x}dx$ (g) $\int 2^x\,dx$ (h) $\int \sin^{-1} x\,dx$

(i) $\int \cos^3 7x\,dx$ (j) $\int \cos^4 7x\,dx$ (k) $\int \frac{x^3}{\sqrt{4-x^2}}dx$ (l) $\int \frac{x}{\sqrt{4-x^2}}dx$

4. Find the limit

(a) $\lim_{x\to\infty} \frac{x^3}{e^{2x}}$ (b) $\lim_{x\to\infty} e^{-x}\ln x$ (c) $\lim_{x\to 0^+} e^{-x}\ln x$ (d) $\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^x$

5. Determine whether the function $f(x)$ is one-to-one. If it is, give a formula for $f^{-1}(x)$. If it isn't, find specific values $x_1 \neq x_2$ for which $f(x_1) = f(x_2)$.

(a) $f(x) = \frac{x+2}{x-2}$ (b) $f(x) = x^4 - 7$ (c) $f(x) = \frac{e^x+2}{e^x}$

6. For what values of x is $f(x)$ increasing?

(a) $f(x) = xe^{3x-1}$ (b) $f(x) = x\ln x$

7. Bacteria in a culture grow at a rate proportional to its size. The count in the culture was 400 after 2 hours and 25,600 after 6 hours.

(a) What was the initial population of the culture?
(b) Find an expression for the population after t hours.
(c) How long does it take for the population to double?