

1. Find the value

(a) $e^{3 \ln 2}$ (b) $\ln e^{-0.3}$ (c) $\log_{10} \left(\frac{1}{100} \right)$ (d) $\log_{100} \left(\frac{1}{10} \right)$
 (e) $(e^{\log_3 7})^{\ln 3} - e^{2 \ln 2} + \log_2 0.125 - \log_{17} 1$
 (f) $\sin^{-1} \left(\sin \left(-\frac{\pi}{5} \right) \right)$ (g) $\cos^{-1} \left(\cos \left(-\frac{\pi}{5} \right) \right)$ (h) $\sin(\sin^{-1}(-.6))$
 (i) $\cos(\sin^{-1}(-.6))$ (j) $\lim_{x \rightarrow 1} \frac{x-1}{x^2+1}$ (k) $\lim_{x \rightarrow 0} \frac{e^{2x}-1}{\sin 3x}$ (l) $\lim_{x \rightarrow \infty} \tan^{-1}(x^3 - 2x)$
 (m) $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$ (n) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^{x^2}$ (o) $\lim_{x \rightarrow 1^+} (x-1) \tan \frac{\pi x}{2}$
 (p) $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+1}}$ (q) $\lim_{x \rightarrow 0} \cos x^{\csc x}$ (r) $\lim_{x \rightarrow \infty} \ln(x+1) - \ln(x/2)$
 (s) $\lim_{x \rightarrow 0^-} \frac{1}{x} - \frac{1}{x^2}$ (t) $\lim_{x \rightarrow 0} x \cot x$

2. Find $\frac{dy}{dx}$

(a) $y = e^{3x^2} - 7$ (b) $y = \ln(7x^3 + 2)$ (c) $y = \tan^{-1}(2x + 5)$ (d) $y = \pi^x + x^\pi$
 (e) $y = \log_2 x$ (f) $y = \frac{(x^2 + 7)^3 e^{7x}}{x^{4/3}(3x + 1)}$ (g) $y = (x^2 + 1)^{2x+7}$ (h) $y = x \sin^{-1}(e^{2x})$

3. Evaluate

(a) $\int x e^{3x^2} dx$ (b) $\int \frac{x^2}{2-7x^3} dx$ (c) $\int_5^8 \frac{dx}{3-x}$ (d) $\int \frac{1}{4+x^2} dx$
 (e) $\int_e^{e^4} \frac{1}{x \sqrt{\ln x}} dx$ (f) $\int 2^x dx$ (g) $\int \frac{x^2 dx}{1+x^6}$
 (h) $\int_1^e x^3 \ln x dx$ (i) $\int \tan^{-1} x dx$ (j) $\int \sin^3(2x) dx$
 (k) $\int \frac{dx}{(1-x^2)^{3/2}}$ (l) $\int \cos^2 \left(\frac{x}{4} \right) dx$
 (m) $\int_0^1 \sqrt{x^2+1} dx$ (n) $\int e^t \sqrt{9-e^{2t}} dt$

4. Let $f(x) = e^{6x} + e^x - 1$. Show that $f(x)$ is one-to-one. Let $g(x)$ be the inverse function of $f(x)$. Determine $g'(1)$.
5. Find the equation of a curve in the xy -plane that contains the point $(0, 3)$ and whose slope at the point xy is $2yx$.
6. A tank contains 1000 L of brine with a concentration of 0.2 kg of salt per liter. In order to dilute the solution, pure water is run into the tank at a rate of 20 L/min and the resulting solution, which is stirred continuously, runs out at the same rate.
 - (a) How many kilograms of salt will remain after half an hour?
 - (b) When will the concentration be reduced to 0.1 kg of salt per liter?