4. $f'(x) = 6e^{6x} + e^x > 0$, so f is increasing and hence one-to-one (Theorem 2, p. 345). Note that $f(0) = 1$, so $g(1) = 0$.

$$g'(1) = \frac{1}{f'(g(1))} = \frac{1}{f'(0)} = 1/(6+1) = 1/7$$

5. Given: $y = f(x)$ is a function with $y' = 2yx$ or $\frac{y'}{y} = 2x$.

Then $\ln|y| = \int \frac{dy}{y} = \int \frac{y'}{y}\,dx = \int 2x\,dx = x^2 + C$, so $|y| = e^{x^2+C} = e^C \cdot e^{x^2}$.

[The right hand side is continuous and near zero, so we can write]

$$y = A \cdot e^{x^2}, \text{ where } 3 = y(0) = Ae^{0^2} = A, \text{ so}$$

$$y = 3e^{x^2}.$$

6. Let $y(t)$ be the amount of salt at time t.

Then $y(0) = 1000 \cdot 0.2$ kg $= 200$ kg and

$$\frac{dy}{dt} = -\left(\frac{y(t)}{1000}\ \frac{kg}{L}\right)\cdot\left(20\ \frac{L}{min}\right) = -\frac{1}{50}\,y\ \frac{kg}{min},$$

so $y(t) = y(0)e^{-t/50} = 200\,e^{-t/50}$

(a) $y(30) = 200\,e^{-30/50} = 200\,e^{-0.6} \approx 110$ kg

(b) We have 0.1 kg/L when there is 100 kg of salt in solution:

$100 = 200\,e^{-t/50} \Rightarrow t = 50 \ln 2 \approx 35$ min