

No books, notes or calculators. Do all work in the blue book, show your work and give reasons when appropriate. Be sure to sign your blue book.

1. (15 points)

(a) Give the form of the partial fraction decomposition of  $f(x)$  without solving for the constants, where  $f(x) = \frac{x^4 + 3x - 1}{(x + 3)^3(x^2 + 9)^2}$ .

(b) Integrate  $\int \frac{x + 6}{(x - 4)(x - 3)} dx$ .

2. (a) (12 points) Evaluate the improper integral  $\int_{-2}^2 \frac{dx}{x^4}$ , or show that it diverges.

3. (16 points) Determine convergence or divergence and evaluate the limits that exist. Indicate whenever you apply L'Hôpital's rule.

(a)  $\lim_{k \rightarrow \infty} \frac{k^3}{e^{2k}}$       (b)  $\lim_{x \rightarrow \infty} \left[ (x^3 + 1)^{\frac{1}{\ln x}} \right]$

4. (7 points) Calculate the Taylor (Maclaurin) polynomial  $P_3(x)$  for the function  $f(x) = (1 + x)^{\frac{1}{2}}$  based at 0.

5. (10 points)

(a) Find a formula for the remainder  $R_2(x)$  for the Taylor polynomial of order 2 based at 1 for the function  $f(x) = \ln x$ .

(b) Obtain a good bound for  $\left| R_2 \left( \frac{1}{2} \right) \right|$ .

6. (8 points) Find  $\sum_{k=1}^{\infty} \left( -\frac{1}{3} \right)^{k+1}$ .

7. (32 points) Determine whether each series converges or diverges. Indicate clearly the name of each test that you use, and give sufficient reasons. If you do the same series more than one way, only the first attempt that has not been crossed out will be graded.

(a)  $\sum_{k=1}^{\infty} \left( \frac{k+1}{10k+1} \right)^2$       (b)  $\sum_{k=1}^{\infty} \frac{k}{\sqrt{4k^3 - 3}}$

(c)  $\sum_{k=1}^{\infty} \frac{(k+2)!}{k!3^k}$       (d)  $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$