Review 2 Fall 1988

1. Give the **form** of the partial fraction decomposition for $f(x)$ **without solving for the constants:**

$$f(x) = \frac{x^4 + 2x^3 + 6x}{(x-2)^2(x+2)(x^2+4)^2}$$

2. Compute the following integrals:

a) $\displaystyle\int \frac{dx}{x^2 - 2x + 3}$ b) $\displaystyle\int \frac{x+3}{x^2+x+1}\,dx$ c) $\displaystyle\int \frac{x+3}{x^2-4}\,dx$

d) $\displaystyle\int \frac{2x^2+9x+12}{(x+2)(x+3)^2}\,dx$ e) $\displaystyle\int \frac{x^3 dx}{x^2-x-6}$ f) $\displaystyle\int \frac{3x^3+x^2-3}{x^2(x^2+3x+3)}\,dx$

3. Use Simpson's rule, with $n = 4$, to write a sum which approximates the integral

$$\int_2^6 \sqrt{x^2-1}\,dx.$$

4.

a) Convert to Cartesian coordinates and identify the curve $r = 4\cos\theta - 2\sin\theta$.
b) Find the equation in polar coordinates for the curve whose cartesian equation is $4x^2 + 9y^2 = 1$.

5. Graph the curve whose equation is $r = 4\sin 3\theta$

6. Sketch the following curves by eliminating the parameter t, and label the direction of increasing t:

a) $x = 2\cos t,\quad y = 5\sin t \quad 0 \le t \le 2\pi$
b) $x = \sqrt{t},\quad y = 3t + 5 \quad t \ge 0.$

7. Set up (but do not even attempt to evaluate) the integral for the arc length of the curve in 6(a).

8. Determine whether the given improper integral converges or diverges. If it converges, calculate its value.

a) $\displaystyle\int_0^1 \frac{dx}{x^{\frac{3}{2}}}$ b) $\displaystyle\int_{-\infty}^{+\infty} x^3 e^{-x^4}\,dx$ c) $\displaystyle\int_0^{\frac{\pi}{2}} \tan x\,dx$

d) $\displaystyle\int_0^5 \frac{dx}{\sqrt{25-x^2}}$ e) $\displaystyle\int_0^4 \frac{dx}{(x-3)^{\frac{1}{7}}}$ f) $\displaystyle\int_3^{+\infty} \frac{dx}{x(\ln x)^2}$

9. Compute the following limits:

a) $\displaystyle\lim_{x\to 0} \frac{\sin x - x}{(\sin x)^3}$ b) $\displaystyle\lim_{x\to 0} \frac{\tan 3x}{\ln(1+x)}$

c) $\displaystyle\lim_{x\to 0^+} (\sin x)^{e^x - 1}$ d) $\displaystyle\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$

e) $\displaystyle\lim_{x\to 0^+} \frac{\cot x}{\ln(\sin x)}$ f) $\displaystyle\lim_{x\to +\infty} (\cot x)^{\frac{1}{x}}$