Review 2 Fall 1989

1. Give the **form** of the partial fraction decomposition for $f(x)$ **without solving for the constants:**

$$f(x) = \frac{x^4 + 2x^3 + 6x}{(x-2)^2(x+2)(x^2+4)^2}$$

2. Evaluate:

a) $\displaystyle\int \frac{2x^2 - x + 2}{x^2(x^2+1)}\, dx$ b) $\displaystyle\frac{x^3+2}{x^2-1}\, dx$

c) $\displaystyle\int \frac{dy}{y^2\sqrt{y^2-7}}$ d) $\displaystyle\int \frac{dx}{\sqrt{16+9x^2}}$ e) $\displaystyle\frac{x-1}{\sqrt{2x-x^2}}\, dx$

3. Use Simpson's rule with $n = 4$ to write a sum which approximates the integral

$$\int_2^6 \sqrt{x^2-1}\, dx$$

4.

a) Convert to Cartesian coordinates and identify the curve $r = 4\cos\theta - 2\sin\theta$.
b) Find the equation in polar coordinates for the curve whose cartesian equation is

$$4x^2 + 9y^2 = 1.$$

5. Graph the curve whose equation is $r = 4\sin 3\theta$

6. Sketch the following curves by eliminating the parameter t, and label the direction of increasing t:

a) $x = 2\cos t, \quad y = 5\sin t, \qquad 0 \le t \le 2\pi$
b) $x = \sqrt{t}, \quad y = 3t + 5$

7. Let C be the curve $x = \dfrac{t^2}{2}, \quad y = \dfrac{1}{3}, \quad t \ge 0.$

a) Show that the points $(0, \frac{1}{3})$ and $(8, 9)$ lie on C and find the corresponding values for t.

b) Find the arc length along C from $(0, \frac{1}{3})$ to $(8, 9)$.

c) Find an equation for the tangent line to C at the point where $t = 4$.

d) Find $\frac{d^2y}{dx^2}$ for C in terms of t.

8. Determine convergence or divergence, **and** evaluate the limits which exist. Give reasons, and indicate each time you apply L'Hopital's rule:

a) $\displaystyle\lim_{x\to\infty} (1 + \frac{6}{x})^x$ b) $\displaystyle\lim_{x\to 0+} x\ln(\sin x)$

c) $\displaystyle\lim_{x\to\infty} (1+x)^{\frac{1}{\ln x}}$ d) $\displaystyle\int_0^8 \frac{dx}{(x-4)^2}$