Tufts University

Math 12 — Department of Mathematics — Fall 1996

Exam 2 Review

1. Determine whether each of the following integrals converges or diverges. If it converges, find its value.

$$\text{a)} \int_0^8 \frac{1}{x^{2/3}}dx \quad \text{b)} \int_0^3 \frac{dx}{(x-1)^4} \quad \text{c)} \int_1^e \frac{1}{x(\ln x)^2}dx \quad \text{d)} \int_1^\infty \frac{x\,dx}{1+x^2}$$

2. Determine whether the following sequences converge or diverge. Justify your answers.

$$\text{a)} \left\{\frac{3n^2+4n}{2n^2+n+5}\right\}_{n=1}^\infty \quad \text{b)} \left\{2+\left(\frac{6}{5}\right)^n\right\}_{n=1}^\infty \quad \text{c)} \left\{\frac{\cos n}{e^n}\right\}_{n=1}^\infty$$

$$\text{d)} 1-\frac{1}{2}, \frac{1}{2}-\frac{1}{3}, \frac{1}{3}-\frac{1}{4}, \frac{1}{4}-\frac{1}{5}, \ldots \qquad \text{e)} 1\sin 1, 2\sin\frac{1}{2}, 3\sin\frac{1}{3}, 4\sin\frac{1}{4}, \ldots$$

$$\text{f)} \left\{(-1)^n\left(\frac{2n+3}{n+5}\right)\right\}_{n=1}^\infty \quad \text{g)} \left\{\left(1+\frac{2}{n}\right)^n\right\}_{n=1}^\infty \quad \text{h)} \left\{\frac{\sin n+4}{n^3}\right\}_{n=1}^\infty$$

3. Determine whether each of the following series converges or diverges. If a series converges find its sum. Justify your answers by identifying any tests you use.

$$\text{a)} \sum_{k=2}^\infty \frac{1}{k^2+k} \quad \text{b)} -\frac{5}{3}+\frac{5}{9}-\frac{5}{27}+\frac{5}{81}+\ldots \quad \text{c)} \sum_{n=1}^\infty (\frac{3}{4})^{n-1}$$

$$\text{d)} \sum_{n=1}^\infty \frac{n+1}{7n} \qquad \text{e)} \sum_{k=1}^\infty \frac{5^{k-1}}{6^{k+2}} \qquad \text{f)} \sum_{n=1}^\infty 2^{n+1}\pi^{-n}$$

4. Integrate

$$\text{a)} \int \frac{3x+2}{x^2-1}dx \quad \text{b)} \int \frac{x^2}{(x+1)^3}dx \quad \text{c)} \int \frac{x^2-2x-1}{(x-1)^2(x^2+1)}dx \quad \text{d)} \int \frac{dx}{x-\sqrt[3]{x}} \quad \text{e)} \frac{dx}{\sqrt{x}+\sqrt[4]{x}}$$

5. True or False

   a) If $\lim_{n\to\infty} b_n = 0$ then $\sum_{n=1}^\infty b_n$ converges.

   b) A decreasing sequence of positive terms converges.

   c) If $\{a_n\}_{n=1}^\infty$ is an increasing, bounded sequence of positive terms, then $\sum_{n=1}^\infty a_n$ converges.

   d) If $|r| \leq 1$, $\sum_{n=1}^\infty ar^{n-1}$ converges.