Math 12 Department of Mathematics Spring 1996

Exam 2 Review

1. Integrate

   (a) $\int \sec^4 \omega t \tan \omega t \, dt$ (b) $\int s\sqrt{20s - s^2}\, ds$ (c) $\displaystyle\int \frac{dx}{(9-x^2)^{3/2}}$

   (d) $\int \sin^2 x \cos^5 x \, dx$ (e) $\displaystyle\int \frac{dx}{\sqrt{x^2+4x+8}}$ (f) $\displaystyle\int \frac{2x^2+3}{x(x-1)^2}\, dx$ (g) $\displaystyle\int \frac{dx}{x^2-4}$

2. Estimate $\ln 3$ using four subintervals and
   (a) Simpson's Rule (S_4) (b) the Trapezoidal Rule (T_4).

3. State whether or not the following improper integrals converge and evaluate those that are convergent.

   (a) $\displaystyle\int_e^\infty \frac{dx}{x \ln x}$ (b) $\displaystyle\int_{-\infty}^\infty \frac{e^x\, dx}{1+e^{2x}}$ (c) $\displaystyle\int_0^\infty \frac{x\, dx}{1+x^2}$

   (d) $\displaystyle\int_0^8 \frac{1}{x^{2/3}}\, dx$ (e) $\displaystyle\int_0^3 \frac{dx}{(x-1)^4}$ (f) $\displaystyle\int_1^e \frac{1}{x \ln^2 x}\, dx$

4. Solve the following differential equations

   (a) $\dfrac{dy}{dt} = 4 - y^2$, given that $y(0) = 0$. (b) $\dfrac{dy}{dt} = e^{y+t}$, given that $y(0) = -1$.

5. Use mathematical induction to prove

   (a) For $n \geq 4 \quad n! > 2^n$ (b) $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.

6. Determine whether the following sequences converge or diverge. Justify your answers.

   (a) $\left\{\dfrac{3n^2+4n}{2n^2+n+5}\right\}_{n=1}^\infty$ (b) $\left\{2 + \left(\dfrac{6}{5}\right)^n\right\}_{n=1}^\infty$ (c) $\left\{\dfrac{\cos n}{e^n}\right\}_{n=1}^\infty$

   (d) $1 - \dfrac{1}{2}, \dfrac{1}{2} - \dfrac{1}{3}, \dfrac{1}{3} - \dfrac{1}{4}, \dfrac{1}{4} - \dfrac{1}{5}, \ldots$ (e) $1\sin 1, 2\sin\dfrac{1}{2}, 3\sin\dfrac{1}{3}, 4\sin\dfrac{1}{4}, \ldots$

   (f) $\left\{(-1)^n\left(\dfrac{2n+3}{n-5}\right)\right\}_{n=6}^\infty$ (g) $\left\{\left(1+\dfrac{2}{n}\right)^n\right\}_{n=1}^\infty$

7. Consider the curve given by parametric equations

$$x = e^t, \quad y = 4 - e^{2t}, \quad -\infty < t \leq 0.$$

   Eliminate the parameter and sketch the curve.

8. Find the length of the curve given by parametric equations

$$x = t\cos t, \quad y = t\sin t, \quad 0 \leq t \leq \pi.$$