Tufts University

Math 12 | Department of Mathematics | Spring 1997

Exam 2 Review

1. Find the limit or show that it doesn't exist. If it is ∞ or $-\infty$, say so.

$a)\ \lim_{x\to\infty}\left(\frac{x-1}{x}\right)^x \qquad b)\ \lim_{x\to 0}\left(\frac{1}{e^x-1}-\frac{1}{x}\right) \qquad c)\ \lim_{x\to\infty}\frac{\ln\ln x}{\sqrt{x}}$

$d)\ \lim_{x\to\infty}\frac{\tan 7x}{x} \qquad e)\ \lim_{x\to 1}\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \qquad f)\ \lim_{x\to 0}(\sin x)^{\tan x}$

2. Evaluate:

$a)\ \int (2x^2+1)e^x\ dx \qquad b)\ \int (2x+1)e^{x^2+x+1}\ dx \qquad c)\ \int (x+1)\ln x\ dx$

$d)\ \int \sin(\ln x)\ dx \qquad e)\ \int_0^{1/2} \sin^{-1} x\ dx \qquad f)\ \int_1^4 e^{\sqrt{x}}\ dx$

$g)\ \int \frac{1-\sin x}{\cos x}\ dx \qquad h)\ \int_0^{\pi/2} \sin^2 x\cos^2 x\ dx \qquad i)\ \int \sin^2 x\cos^3 x\ dx$

3. Integrate:

$a)\ \int \frac{3x-2}{x^2-4}\ dx \qquad b)\ \int \frac{x^3}{(x+1)^2}\ dx \qquad c)\ \int \frac{x+6}{(x^2+4)(x-2)}\ dx$

$d)\ \int \sqrt{2-x^2}\ dx \qquad e)\ \int \sqrt{e^{2x}-9}\ dx \qquad f)\ \int_0^3 x^2\sqrt{9-x^2}\ dx$

$g)\ \int \frac{1}{x\sqrt{x+1}}\ dx \qquad h)\ \int \frac{x^3}{(x+1)^2}\ dx \qquad i)\ \int \sqrt{\frac{1-x}{x}}\ dx$

4. Determine whether each of the following integrals converges:

$a)\ \int_0^1 \frac{dx}{\sqrt{1-x}} \qquad b)\ \int_0^3 \frac{dx}{(x-2)^2} \qquad c)\ \int_{-\infty}^{+\infty} xe^{-x^2}\ dx$

$d)\ \int_1^\infty \frac{dx}{x^3} \qquad e)\ \int_1^\infty \frac{dx}{x^3+1} \qquad f)\ \int_1^\infty \frac{\sin x}{x^3+1}\ dx$

5. Determine whether each of the following sequences converges:

$a)\ \left\{\frac{2n+1}{3n-1}\right\}_{n=1}^\infty \qquad b)\ \left\{(-1)^n\frac{n^2-1}{n^2+n+1}\right\}_{n=1}^\infty \qquad c)\ \left\{(-1)^{n+1}\frac{2n-7}{n^2+3}\right\}_{n=1}^\infty$

$d)\ \left\{1+\frac{3^n}{2^{n-2}}\right\}_{n=1}^\infty \qquad e)\ \left\{\frac{\cos n-7}{n^2+1}\right\}_{n=1}^\infty \qquad f)\ \left\{\left(1+\frac{1}{2n}\right)^{3n}\right\}_{n=1}^\infty$