

1. Find the following limits or state that the limit does not exist (if the limit is ∞ or $-\infty$ you should say so).

(a) $\lim_{x \rightarrow 1} \frac{x-1}{x^2+1}$

(b) $\lim_{x \rightarrow 0} \frac{e^{2x}-1}{\sin 3x}$

(c) $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$

(d) $\lim_{x \rightarrow 0} \frac{\sin x}{x^2}$

(e) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{x^2}$

(f) $\lim_{x \rightarrow 1^+} (x-1) \tan \frac{\pi x}{2}$

2. Evaluate each integral

(a) $\int_1^e x^3 \ln x \, dx$

(b) $\int \tan^{-1} x \, dx$

(c) $\int \sin^3(2x) \, dx$

(d) $\int \frac{dx}{(1-x^2)^{\frac{3}{2}}}$

(e) $\int \frac{x^2+2}{x^2+1} \, dx$

(f) $\int \cos^2\left(\frac{x}{4}\right) \, dx$

(g) $\int_0^1 \sqrt{x^2+1} \, dx$

(h) $\int e^t \sqrt{9-e^{2t}} \, dt$

(i) $\int \frac{\sqrt{x}}{x+1} \, dx$

(j) $\int \frac{x+3}{x^2-x} \, dx$

(k) $\int \frac{x-1}{x^2-2x+10} \, dx$

(l) $\int \frac{\cos x \, dx}{\sqrt{4 \sin x - \sin^2 x}}$

3. Set up the form of the partial fraction decomposition. Do not evaluate the integrals.

(a) $\frac{5}{x^2(x^2-16)(x^2+x+6)}$

(b) $\frac{4x^2+5x+7}{4x^2+8x+3}$

(c) $\frac{x^2}{(x^2+1)^2(x-1)}$

4. Use the Midpoint Rule, the Trapezoidal Rule and Simpson's Rule each with $n = 4$ to estimate the integral $\int_0^4 \frac{dx}{1+x}$. Leave each of your answers as a (numerical) sum.

5. Textbook, p. 487 34

6. Determine whether the following integrals are convergent or divergent. Evaluate those that are convergent.

(a) $\int_1^{\infty} \frac{1}{(x+1)^4} \, dx$

(b) $\int_{-1}^3 \frac{1}{x^4} \, dx$

(c) $\int_0^{\infty} x e^{-x^2} \, dx$

(d) $\int_2^{\infty} \frac{x}{\sqrt{x^3-1}} \, dx$