

# Math 12 Solutions to review sheet

① a)  $\int \frac{\ln(\ln x)^4}{x} dx = \int \ln u^4 du = u \ln u^4 - \int u \frac{4u^3}{u^4} du = u \ln u^4 - 4u + C$

$\uparrow$   
 $\ln x = u$   
 $\frac{1}{x} dx = du$

parts  $u_1 = 1$   $v_1 = u$   
 $u_1 = \ln u^4$   $du_1 = \frac{4u^3}{u^4} du$

$= \ln x \ln(\ln x)^4 - 4 \ln x + C$

b)  $\int e^x - 3e^{3x} dx = \int e^x \cdot e^{-3e^x} dx = -\frac{1}{3} \int e^u du = -\frac{1}{3} e^u + C = -\frac{1}{3} e^{-3e^x} + C$

$\uparrow$   
 $-3e^x = u$   
 $-3e^x dx = du$

$-\frac{1}{3} e^{-3e^x} + C$

c)  $\int (\sqrt{x-2})^3 dx = \int 2u u^3 du = 2 \int u^4 du = 2 \frac{u^5}{5} + C = \frac{2(\sqrt{x-2})^5}{5} + C$

$\uparrow$   
 $\sqrt{x-2} = u$   
 $\frac{1}{2\sqrt{x-2}} dx = du \Rightarrow dx = 2\sqrt{x-2} du = 2u du$

$\frac{2(\sqrt{x-2})^5}{5} + C$

d)  $\int x \sqrt{x+2} dx = 2 \int (u^2-2) u^2 du = 2 \int (u^4 - 2u^2) du = 2 \left( \frac{u^5}{5} - 4 \frac{u^3}{3} \right) + C =$

$\sqrt{x+2} = u \Rightarrow x = u^2 - 2$   
 $\frac{1}{2\sqrt{x+2}} dx = du \Rightarrow dx = 2\sqrt{x+2} du$

$= \frac{2(\sqrt{x+2})^5}{5} - \frac{4}{3} (\sqrt{x+2})^3 + C$

e)  $\int \frac{\sqrt{x}}{x+1} dx = \int \frac{u \cdot 2u du}{u^2+1} = \int \frac{2u^2}{u^2+1} du = \int \frac{2[(u^2+1) - 1]}{u^2+1} du =$

$\uparrow$   
 $u = \sqrt{x}$   
 $\frac{1}{2\sqrt{x}} dx = du \Rightarrow dx = 2\sqrt{x} du$

$= 2 \int \left( 1 - \frac{1}{u^2+1} \right) du = 2u - 2 \tan^{-1} u + C = 2\sqrt{x} - 2 \tan^{-1} \sqrt{x} + C$

$2\sqrt{x} - 2 \tan^{-1} \sqrt{x} + C$