

$$c) \frac{x^2}{(x^2+1)^2(x-1)} = \frac{a}{x-1} + \frac{bx+c}{x^2+1} + \frac{dx+e}{(x^2+1)^2} = \frac{a(x^2+1)^2 + (bx+c)(x-1)(x^2+1) + (dx+e)(x-1)}{(x-1)(x^2+1)^2}$$

Making $x=1$ get $4a=1 \Rightarrow a=\frac{1}{4}$
 Writing out the numerator in the fraction, get $a(x^4+2x^2+1) + (bx+c)(x^3-x^2+x-1) + dx^2+ex-dx+e = x^2$

$$(a+b)x^4 + (c-b)x^3 + (2a-c+b+d)x^2 + (c-b+e-d)x + a-c-e = x^2$$

$$a+b=0 \Rightarrow b=-a = -\frac{1}{4}$$

$$c-b=0 \Rightarrow c=b = -\frac{1}{4}$$

$$2a-c+b+d=1 \Rightarrow d=1$$

$$a-c-e=0 \Rightarrow e=\frac{1}{2}$$

$$\frac{x^2}{(x^2+1)^2(x-1)} = \frac{1}{4(x-1)} - \frac{x+1}{4(x^2+1)} + \frac{x+1}{2(x^2+1)^2}$$

$$③ \text{ Midpoint rule } \approx \frac{1}{3} \left(\frac{1}{1+\frac{1}{2}} + \frac{1}{1+\frac{3}{2}} + \frac{1}{1+\frac{5}{2}} + \frac{1}{1+\frac{7}{2}} \right)$$

$$\text{Trapezoidal rule } \approx \frac{1}{2} \left[\frac{1}{1+0} + 2 \frac{1}{1+1} + 2 \frac{1}{1+2} + \frac{2}{1+3} + \frac{1}{1+4} \right]$$

$$\text{Simpson's rule } \approx \frac{1}{3} \left[\frac{1}{1+0} + 4 \frac{1}{1+1} + 2 \frac{1}{1+2} + 4 \frac{1}{1+3} + \frac{1}{1+4} \right]$$

$$\text{Error } f'(x) = \frac{-1}{(1+x)^2} \quad f''(x) = \frac{2}{(1+x)^3} \quad f'''(x) = \frac{-6}{(1+x)^4} \quad f^{(4)}(x) = \frac{24}{(1+x)^5}$$

$$f''(x) \leq 2 \text{ in the interval } (0, 4)$$

$$f^{(4)}(x) \leq 24 \text{ in the interval } (0, 4)$$

$$E_M \leq \frac{2 \cdot 4^3}{24 \cdot 4^2} = \frac{1}{3}$$

$$E_T \leq \frac{2 \cdot 4^3}{12 \cdot 4^2} = \frac{2}{3}$$

$$E_S \leq \frac{24 \cdot 4^5}{180 \cdot 4^4 \cdot 45} = \frac{24 \cdot 0.5}{180 \cdot 45}$$

p. 487 ③④ $V = \int A dx$

$$V \approx \frac{1}{3} [0.68 + 4 \cdot 0.65 + 2 \cdot 0.64 + 4 \cdot 0.61 + 2 \cdot 0.58 + 4 \cdot 0.59 + 2 \cdot 0.53 + 4 \cdot 0.55 + 2 \cdot 0.52 + 4 \cdot 0.50 + 0.48]$$