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p. 512 (30) $x = \text{kg of salt in the tank}$

$$\frac{dx}{dt} = 5 \cdot 0.05 + 10 \cdot 0.04 + \frac{15x}{1000} = 0.25 + 0.4 - 0.015x = 0.65 - 0.015x$$

$$\frac{dx}{0.65 - 0.015x} = dt \quad - \frac{\ln |0.65 - 0.015x|}{0.015} = t + c$$

$$0.65 - 0.015x = ce^{-0.015t}$$

$$x = \frac{0.65 - ce^{-0.015t}}{0.015}$$

$$0 = x(0) = \frac{0.65 - c}{0.015} \Rightarrow c = 0.65 \cdot 0.015 = 0.00975$$

$$x = \frac{0.65 - 0.00975e^{-0.015t}}{0.015} = \boxed{\frac{0.65}{0.015} - 0.65e^{-0.015t} = x}$$

5 a) $\int \frac{dx}{(x+1)^4} = -\frac{1}{3} \frac{1}{(x+1)^3} + c$

$$\int_1^6 \frac{1}{(x+1)^4} dx = \lim_{M \rightarrow 6^-} \int_1^M \frac{1}{(x+1)^4} dx = \lim_{M \rightarrow 6^-} \left[-\frac{1}{3(x+1)^3} \right]_1^M = \lim_{M \rightarrow 6^-} \frac{1}{3 \cdot 8} - \frac{1}{3 \cdot 2^3} = \frac{1}{24}$$

b) $\int_{-1}^3 \frac{1}{x^4} dx = \lim_{m \rightarrow 0^-} \int_{-1}^m \frac{1}{x^4} dx + \lim_{m \rightarrow 0^+} \int_m^3 \frac{1}{x^4} dx = \lim_{m \rightarrow 0^-} \left[-\frac{x^{-3}}{3} \right]_{-1}^m + \lim_{m \rightarrow 0^+} \left[-\frac{x^{-3}}{3} \right]_m^3$

$\lim_{m \rightarrow 0} \frac{1}{3m^3}$ does not exist. So the integral diverges

c) $\int x e^{-5x} dx = \frac{x e^{-5x}}{-5} + \int e^{-5x} dx = -\frac{x e^{-5x}}{5} + \frac{1}{25} e^{-5x} + c$

$u = x \quad du = 1$
 $dV = e^{-5x} \quad v = \frac{e^{-5x}}{-5}$

$$\lim_{m \rightarrow \infty} m e^{-5m} = \lim_{m \rightarrow \infty} \frac{m}{e^{5m}} = \lim_{m \rightarrow \infty} \frac{1}{5e^{5m}} = 0$$

$$\int_0^{\infty} x e^{-5x} dx = \lim_{m \rightarrow \infty} \int_0^m x e^{-5x} dx = \lim_{m \rightarrow \infty} \left[-\frac{x e^{-5x}}{5} + \frac{1}{25} e^{-5x} \right]_0^m =$$