Review 3

1. Decide whether the following statements are True (T) or False (F). On the inside cover of your blue book, list the letters (a) through (f) and put a T or F next to each one. **Answers only** will be graded.

   a) By L'Hopital's Rule $\lim_{x\to 1} \dfrac{3x^2+2x-5}{2x^2-3} = \lim_{x\to 1} \dfrac{6x+2}{4x} = 2.$

   b) If $\lim_{k\to\infty} b_k = 0$ then $\sum_{k=1}^{\infty} b_k$ converges.

   c) The series $\dfrac{1}{100} + \dfrac{1}{101} + \dfrac{1}{102} + \cdots$ converges.

   d) The series $\sum_{k=1}^{\infty} \dfrac{3^{k+1}}{4^{k+2}}$ converges.

   e) The series $\sum_{k=1}^{\infty} \dfrac{1}{(1+k)[\ln(1+k)]^2}$ converges. (You may assume that the improper integral $\int_1^{\infty} \dfrac{dx}{(1+x)[\ln(1+x)]^2}$ converges.)

   f) $\left| \sum_{k=27}^{\infty} (-1)^{k+1} \dfrac{1}{\sqrt[3]{k}} \right| \le \dfrac{1}{3}.$

2. Find the limits

   a) $\lim_{x\to 0^-} (1-x)^{\frac{2}{x}}$

   b) $\lim_{x\to 0} \dfrac{(1+x)^{\frac{1}{3}} - 1 - \frac{1}{3}x}{x^2}$

3. Decide whether the following integrals converge or diverge and evaluate the limits which exist.

   a) $\int_0^1 x^{-\frac{4}{3}}\, dx$

   b) $\int_0^{\infty} \dfrac{dx}{1+x^2}$

4. Determine convergence or divergence for each series below. Indicate clearly the name(s) of the test(s) you are using. Cross out all irrelevant work and circle the work to be graded. If a problem has been done several different ways, **only the first attempt** that has not been crossed out will be graded.

   a) $\sum_{k=1}^{\infty} \dfrac{k^2+9}{2k^4-k+3}$

   b) $\sum_{k=1}^{\infty} \dfrac{|\cos k|}{k^2}$

   c) $\sum_{k=1}^{\infty} \dfrac{2k}{e^k}$

   d) $\sum_{k=1}^{\infty} \dfrac{\ln k}{k^3}$

5. For each series below determine whether it converges absolutely, converges conditionally, or diverges.

a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}+1}$ b) $\displaystyle\sum_{k=1}^{\infty} (-1)^k \left(\frac{1}{2}+\frac{1}{k}\right)^k$

6. The first swing of the bob of a pendulum is 5 inches. In each subsequent swing the bob travels $\frac{2}{3}$ of the preceding swing. Find how far the bob will travel before coming to rest.

7. Let $S = \displaystyle\sum_{1}^{\infty} a_k$ and suppose the partial sums of this series are given by

$$S_k = 1 + \frac{\sin 2k}{k}.$$

What is S?

8. The first three non-zero terms of the Taylor Series for

$$\left(\sum_{k=1}^{\infty} \frac{x^k}{2k}\right)\left(\sum_{k=0}^{\infty} \frac{6}{k!}x^k\right)$$

are

a) $3 + \frac{9}{2}x + 4x^2$ b) $3x + \frac{9}{2}x^2 + 4x^3$ c) $1 + 3x + \frac{9}{2}x^2$

d) $\frac{13}{2}x + \frac{51}{4}x^2 + \frac{94}{6}x^3$ e) none of these

9. The series $\displaystyle\sum_{k=0}^{\infty} \frac{k^{2k}}{(2k)!}$

a) converges by the ratio test
b) diverges
c) converges by the root test
d) converges by the comparison test
e) none of these

10. The series $\displaystyle\sum_{k=20}^{\infty} \frac{1}{\sqrt{4k+8}}$

a) converges by the integral test
b) converges by the limit comparison test
c) diverges by the limit comparison test
d) diverges by the ratio test
e) none of these

11. $\lim_{x\to\infty}\left(1+\frac{1}{x}+\frac{1}{x^2}\right)^x$

a) does not exist
b) equals e
c) equals e^{-1}
d) equals e^2
e) none of these

12. $\left\{\frac{1}{n}+\frac{1}{n+1}+\cdots+\frac{1}{n+6}\right\}_{n=1}^{\infty}$ is

a) an infinite series which converges
b) an infinite series which diverges
c) a sequence which does not converge to 0
d) a sequence which converges to 0
e) none of these

13.

a) Let $f(x)$ be a function. For the Taylor series of $f(x)$ about a, write the general formula for $R_n(x)$, the Lagrange form of the remainder.

b) Find the Taylor series for e^{2x} about $a = \ln 3$. Express your answer in $\sum$ notation.

c) Find $R_5(\frac{1}{2}+\ln 3)$ for the particular series in part (b).

14. Let $f(x) = \sum_{k=2}^{\infty}(-1)^k \frac{(3x)^k}{\ln k}$

a) Determine the interval of convergence of this series. Show all work.

b) Let $P_4(x)$ denote the Maclaurin polynomial for f whose highest power of x is x^4. Let u be the smallest upper bound for $|f(\frac{1}{6}) - P_4(\frac{1}{6})|$, i.e., number u such that $|f(\frac{1}{6} - P_4(\frac{1}{6})| \le u$, that you can get by the methods we have studied.

Select the letter from the following list that correctly describes u.
(Answer only will be graded)

a) $0 \le u \le \frac{1}{36\ln 5}$
b) $\frac{1}{36\ln 5} < u \le \frac{1}{30\ln 5}$
c) $\frac{1}{30\ln 5} < u \le \frac{1}{20\ln 5}$
d) $\frac{1}{20\ln 5} < u \le \frac{1}{15\ln 5}$
e) $u > \frac{1}{15\ln 5}$

15. By integrating an appropriate series, find the Maclaurin Series for $\ln(1+4x)$. Express your answer in $\sum$ notation.

16. Calculate the value of $\cos 58^o$ to an accuracy of $\frac{1}{2\cdot 10^4}$