Review 3 Fall 1988

1. Decide whether each statement is true (T) or ridiculous (R). To justify your answer, you should state the reason why, or else give a counterexample.

   a) If $\lim_{k\to\infty} u_k = 0$, then $\sum_{k=1}^{\infty} u_k$ converges.

   b) The series $\frac{1}{101} + \frac{1}{102} + \frac{1}{103} + \cdots$ converges.

   c) The series $\sum_{k=1}^{\infty} \frac{3^{k+1}}{4^{k+2}}$ converges.

   d) The alternating series test can be used to prove absolute convergence.

   e) If $\sum_{k=1}^{\infty} u_k$ converges, then $\sum_{k=1}^{\infty} (-1)^k u_k$ also converges.

   f) The series $\sum_{k=1}^{\infty} \frac{1}{(k+1)[\ln(k+1)]^2}$ converges.

   g) $\left| \sum_{k=27}^{\infty} (-1)^{k+1} \frac{1}{\sqrt[3]{k}} \right| \le \frac{1}{3}$.

2. Determine whether each of the series below converges or diverges. Indicate clearly the name(s) of the test(s) you are using.

   a) $\sum_{k=1}^{\infty} \frac{k^2+9}{2k^4-k+3}$ b) $\sum_{k=1}^{\infty} \frac{\cos k}{k^2}$ c) $\sum_{k=1}^{\infty} \frac{2^k}{e^k}$

   d) $\sum_{k=1}^{\infty} \frac{\ln k}{k^3}$ e) $\sum_{n=1}^{\infty} \frac{n \cos n}{1+n^4}$ f) $\sum_{n=2}^{\infty} \ln\left(1+\frac{1}{n}\right)$

3. For each of the series below, determine whether it converges absolutely, converges conditionally, or diverges.

   a) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}+1}$ b) $\sum_{k=1}^{\infty} (-1)^k \left(\frac{1}{2}+\frac{1}{k}\right)^k$ c) $\sum_{k=1}^{\infty} \frac{(-1)^k k}{k^2+1}$

4. Find the radius and interval of convergence of each of the following series.

   a) $\sum_{n=1}^{\infty} \frac{(x-3)^n}{2^n}$ b) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2 (x+2)^n}{n^3+1}$

5.

a) Determine the third degree Taylor polynomial of $f(x) = \tan x$ at $a = \frac{\pi}{4}$.
b) Write the first four terms of the MacLaurin series of $f(x) = \frac{e^x}{1-x}$. For what values of x does the series represent the function?

6. By manipulating appropriate MacLaurin series, determine the MacLaurin series of the following functions:

a) $\dfrac{\sin x - x}{x^3}$

b) $\dfrac{x^2}{(1-x)^2}$