Review 3 Spring 1990

1. True or False

   a) By L'Hopital's Rule $\lim_{x\to 1} \dfrac{3x^2+2x-5}{2x^2-3} = \lim_{x\to 1} \dfrac{6x+2}{4x} = 2$

   b) If $\lim_{x\to\infty} b_k = 0$ then $\sum_{k=1}^{\infty} b_k$ converges

   c) The series $\dfrac{1}{100} + \dfrac{1}{101} + \dfrac{1}{102} + \cdots$ converges

   d) $\left| \sum_{k=27}^{\infty} (-1)^{k+1} \dfrac{1}{\sqrt[3]{k}} \right| \leq \dfrac{1}{3}$

2. Find the limits

   a) $\lim_{x\to -1} \dfrac{x^2-1}{\ln(4+3x)}$ b) $\lim_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{\cos x - 1} \right)$

3. Determine whether each of the integrals converges or diverges. If it converges, find its value.

   a) $\int_{-1}^{1} x^{-\frac{4}{3}}\, dx$ b) $\int_{0}^{\infty} \dfrac{dx}{1+x^2}$

4. Find the sum of the series $2 - \dfrac{4}{3} + \dfrac{8}{9} - \dfrac{16}{27} + \cdots + (-1)^k \dfrac{2^{k+1}}{3^k} + \cdots$

5. Determine whether each of the series converges or diverges. Indicate clearly the name(s) of the test(s) you are using.

   a) $\sum_{k=1}^{\infty} \dfrac{3k^2+1}{7k^2+9}$ b) $\sum_{k=1}^{\infty} \dfrac{\cos k}{k^{\frac{3}{2}}}$ c) $\sum_{k=1}^{\infty} k(\dfrac{\pi}{4})^k$ d) $\sum_{k=1}^{\infty} \dfrac{\ln k}{k^3}$

   e) $\sum_{k=1}^{\infty} \dfrac{3k}{e^k}$ f) $\sum_{k=1}^{\infty} \dfrac{\sqrt{k}}{2k-3}$ g) $\sum_{k=1}^{\infty} \dfrac{3k+5}{k2^k}$

6. Find the interval of convergence of the series

$$\sum_{k=1}^{\infty} \frac{(-1)^k (x+4)^k}{3^k (1+k)}$$

7. Find the third degree Taylor polynomial $p_3(x)$ for $f(x) = \sin 2x$ about $a = \frac{\pi}{3}$.

8. Let $f(x) = x + \dfrac{1}{(x-1)^2}$

   a) Find $f'(x)$ and $f''(x)$. b) Find $f^{(k)}(x)$ for $k \geq 3$.
   c) Find the Taylor Series for f about $a = 3$.
   Express your answer in summation notation.