Review 3 Fall 1991

1. Find $\lim_{n\to\infty} a_n$ (if it exists)

   a) $a_n = \sqrt[n]{2}$ b) $a_n = 1 + (-1)^n$ c) $a_n = \left(\frac{n+1}{n-1}\right)^n$ d) $a_n = \frac{(3000)^n}{n!}$

2. Find a closed expression for the partial sum s_n and use it to compute the sum of the series
$$2 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \cdots + (-1)^{n-1}\frac{4}{3^{n-1}} + \cdots$$

3. Find the sum of the series
$$\sum_{n=0}^{\infty}\left(\frac{2}{3^n} - \frac{3}{4^n}\right).$$

4. Determine whether the following series converge. Show your tests in detail. Specify the names of the tests you use. Give reasons for the convergence or divergence of any series you use in comparison or limit comparison tests.

   a) $\sum_{n=0}^{\infty} \frac{2n-1}{\sqrt{n^3+5}}$ b) $\sum_{n=0}^{\infty} \frac{2n-1}{n^3+5}$ c) $\sum_{n=0}^{\infty} (-1)^n \frac{2n-1}{3n+5}$

   d) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{\frac{3}{2}}}$ e) $\sum \left(\frac{n}{3n-2}\right)^n$ f) $\sum \frac{5^n}{3n^2+1}$

   g) $\sum_{n=0}^{\infty} \frac{n^3 3^n}{n!}$ h) $\sum \frac{(-1)^n n}{e^n}$

5. Determine (i) the values of x for which the series converge and (ii) the values of x for which the series converge absolutely.

   a) $\sum_{n=0}^{\infty} \frac{(-3)^n(x-1)^n}{n}$ b) $\sum_{n=0}^{\infty} \frac{(-3)^n(x-1)^n}{n^2}$

   c) $\sum_{n=0}^{\infty} \frac{n!(x-1)^n}{(-3)^n}$ d) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{(-3)^n n!}$

6. From our work with geometric series, we know that
$$(*) \quad \frac{1}{1+x} = \sum_{n=0}^{\infty}(-1)^n x^n \quad , \quad |x| < 1.$$

   a) By differentiating $(*)$, find a series expression for $\frac{-1}{(1+x)^2}$.

   b) By integrating $(*)$, find a series expression for $\ln(1+x)$.

7. Find the Taylor series for $f(x) = e^{-2x}$ expanded about $a = 0$.