Review 3 Spring 1991

1. Do the following sequences converge? If so, what is the limit?

   a) $a_n = \dfrac{2n-1}{2n+1}$ b) $a_n = \dfrac{2n-1}{\ln(n)+1}$ c)$a_n = \dfrac{\sqrt{n}-1}{\int_1^n \frac{1}{x}\,dx + 1}$ d) $a_n = \sqrt[n]{n^3}$

   e) $a_n = \left(1 - \dfrac{1}{n^2}\right)^n$ f) $a_n = \dfrac{3^n \cdot 6^n}{2^{-n} \cdot n!}$ g) $a_n = \sqrt[n]{\dfrac{x^n}{2n+1}}$

2. TRUE OR FALSE (and why?!!!)?

   a) If $a_n \to 0$ then $\sum_{n=1}^{\infty} a_n$ converges.

   b) If $\sum a_n$ diverges, for a sufficiently small k, $\sum k a_n$ converges.

   c) $\sum \frac{1}{n}$ diverges but $\sum (-1)^n \frac{1}{n}$ converges.

   d) If $\sum a_n$ converges to A, $\sum b_n$ converges to B, then $\sum \frac{a_n}{b_n}$ converges to $\frac{A}{B}$ provided that $b_n \neq 0, \quad B \neq 0$.

   e) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 0$.

   f) The sequence $a_n = \sum_{k=1}^{n} \frac{1}{k}$ is bounded above.

3. Determine whether the following series converge or not. If so, find the sum if possible. Indicate clearly how you arrived at your conclusion!

   a) $\displaystyle\sum_{k=1}^{\infty} \frac{3k^2+1}{7k^2+9}$ b) $\displaystyle\sum_{k=1}^{\infty} \frac{\cos k}{k^{\frac{3}{2}}}$ c) $\displaystyle\sum_{n=1}^{\infty} n\left(\frac{\pi}{4}\right)^n$ d) $\displaystyle\sum_{k=1}^{\infty} \frac{\ln k}{k^3}$ e) $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{e^n}$

   f) $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k}}{2k-3}$ g) $\displaystyle\sum_{k=1}^{\infty} \frac{3k+5}{k2^k}$ h) $\displaystyle\sum_{k=1}^{\infty} \frac{k^k}{k!}$ i) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n!)^2}{(2n)!}$

4. Estimate the magnitude of the error if the first four terms are used to approximate the series:

$$\sum_{n=0}^{\infty} (-1)^n t^n \quad \text{for} \quad 0 < t < 1$$

5. Show that the sum of the first $2n$ terms of $1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} + \cdots$ is the same as the sum of the first n terms of $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots$.

   Do these series converge? What is the sum of the first $2n+1$ terms of the series? If the series converge, what is their sum?