Final Review

1. Simplify

   a) $e^{3\ln x + 2\ln 5}$ b) $\sin^{-1}\left(\sin\frac{4\pi}{3}\right)$ c) $\cos(\sin^{-1}(1) - \sin^{-1}(-1))$

2. Find x given that $\ln(\frac{1}{x^2}) = e$

3. Find $\frac{dy}{dx}$ if

   a) $y = x(\cos^{-1} x)^2$ b) $y = x^{(x-e^{3x})}$

   c) $y = \dfrac{4^{x+5}}{(x^2+1)^3(x-3)^4}$ (use logarithmic differentiation)

4. Solve the differential equation $\frac{dy}{dx} = 2x(y^2+1)$ with $y(0) = -1$

5. Let

$$\begin{cases} x = t^7 + t^3 - 2 \\ y = 5 + 4t^2 \end{cases}$$

   a) Find $\frac{dy}{dx}$ for $t = 1$ b) Find $\frac{d^2y}{dx^2}$ for $t = 1$

6. Integrate

   a) $\displaystyle\int \frac{\ln x}{x}\,dx$ b) $\displaystyle\int x^2 \sin x\,dx$ c) $\displaystyle\int \sin^3(x+1)\cos^3(x+1)\,dx$

   d) $\displaystyle\int \frac{x\,dx}{\sqrt{x^2+2x+5}}$ e) $\displaystyle\int \frac{2x^2-3x-1}{x^2-2x-3}\,dx$ f) $\displaystyle\int_3^7 \frac{dx}{x^2-6x+25}$

7.

   a) Sketch the curves $r = 1 + \sin\theta$ and $r = 3\sin\theta$ on the same axes.

   b) Convert to rectangular coordinates: $r^2 = \sin 2\theta$.

8. Investigate the following limits

   a) $\lim\limits_{x\to 0} \dfrac{x^2}{\sin x - x}$ b) $\lim\limits_{x\to 1} \dfrac{x + \cos\pi x}{\sin\pi x}$ c) $\lim\limits_{x\to\infty} x^{\left(\frac{1}{x^2+1}\right)}$

9. Determine convergence or divergence and justify your answers

a) $\sum_{n=1}^{\infty} \sqrt{\frac{n-1}{n^3+6}}$ b) $\sum_{n=1}^{\infty} \frac{1}{5n\log(1+n)}$ c) $\sum_{n=1}^{\infty} \frac{8^n}{2\cdot 4\cdot 6\cdots(2n)}$

10. For what values of x does $\sum_{n=1}^{\infty} \frac{(-1)^n(2x+1)^n}{n+1}$

a) converge absolutely b) converge conditionally c) diverge

11. True or False

a) If $\lim_{n\to\infty} a_n = 0$ the series $\sum_{n=0}^{\infty} a_n$ converges.

b) If $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges conditionally.

c) If $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

12. Find the Taylor expansion of $f(x) = \frac{1}{(1+x)^2}$ about $a = 1$ (i.e., in powers of $x-1$).

13. Use the first five non-zero terms of the Taylor series expansion for $\cos x$ about $a = 0$ to find the first four non-zero terms of the series expansion for

$$\int_0^x \frac{1-\cos t}{t^2}\, dt.$$

14. Use the Taylor series for $\sin x$ in powers of x to compute $\sin(0.4)$ to within an accuracy of .0001. Show bounds on your error.

15. Express in the form $x = iy$, x, y real:

a) $\frac{3+i}{1-2i}$ b) $3\text{cis}\frac{\pi}{6}$ c) $e^{2+\frac{\pi i}{3}}$ d) $(1+i)^{10}$

16. Find all complex sixth roots of -64. Leave your answer in the form $r\text{cis}\theta$. $[r\text{cis}\theta = r(\cos\theta + i\sin\theta)]$

17. Compute the radius and center of the circle of convergence of

$$\sum_{n=1}^{\infty} \frac{(z+3-4i)^n}{n^5\cdot 5^n}$$

18. Determine interval and radius of convergence of

$$\sum_{k=1}^{\infty} \frac{(2x)^k}{3k}$$

Show your work.
Sketch the circle in the complex plane labelling axes, center and radius.

19. Determine interval and radius of convergence of

$$\sum_{k=1}^{\infty} \frac{k^2(x+2)^k}{(k+1)!}$$

20. Find Maclaurin Series and radius of convergence of $\dfrac{2x}{2+x^2}$