Final Review Spring 1989

I. MULTIPLE CHOICE OR FILL IN THE BLANK.

In Part I **answers only** will be graded. Write the number 1 through 13 on the inside cover of you blue book and record your answers next to the appropriate numbers.

1. $\sqrt{e^{2\ln 10 - 4\ln 3}}$ equals

   a) 9/10 b) $e^{19/2}$ c) 10/9 d) 19/10 e) 19/2

2. $\dfrac{\ln\left(\frac{1}{16}\right)}{\ln\left(\frac{2}{5}\right) - \ln\left(\frac{4}{5}\right)}$ equals

   a) $\ln\left(\dfrac{1}{16} + \dfrac{2}{5}\right)$ b) $\frac{1}{4}$ c) -4 d) $\ln(\frac{1}{8})$ e) 4

3. $\sin\left(\cos^{-1}\left(-\dfrac{2}{5}\right)\right)$ is

   a) $\frac{3}{5}$ b) $\sqrt{21}/5$ c) $\sqrt{29}/5$ d) $-\sqrt{29}/5$ e) $-\sqrt{21}/5$

4. The graph of $r = 4\cos\theta - 2\sin\theta$ is

   a) a circle through the origin
   b) a circle not through the origin
   c) a straight line through the origin
   d) a straight line not through the origin
   d) a cardioid

5. If $y = (x^2+8)^{\sin x}$ then $\dfrac{dy}{dx} =$

   a) $\sin x(x^2+8)^{\sin x - 1}2x$

   b) $(2x)^{\cos x}[\sin x \ln(x^2+8)]$

   c) $(x^2+8)^{\sin x}\left[\dfrac{2x\sin x}{x^2+8} + \cos x \ln(x^2+8)\right]$

   d) $(x^2+8)^{\sin x}[\sin x \ln(x^2+8)]$

   e) $\sin x(x^2+8)^{\sin x - 1}2x\left[\dfrac{2x\sin x}{x^2+8} + \cos x \ln(x^2+8)\right]$

6. $\dfrac{1-3i}{2+4i}$ equals

   a) $\frac{1}{2} - \frac{3}{4}i$ b) $\frac{7-5i}{10}$ c) $\frac{1+i}{2}$ d) $\frac{-1-i}{2}$ e) $\frac{5-5i}{4}$

7. The first three non-zero terms of the Taylor Series for

$$\left(\sum_{k=1}^{\infty} \frac{x^k}{2k}\right)\left(\sum_{k=0}^{\infty} \frac{6}{k!}x^k\right)$$

are

a) $3 + \frac{9}{2}x + 4x^2$ b) $3x + \frac{9}{2}x^2 + 4x^3$ c) $1 + 3x + \frac{9}{2}x^2$

d) $\frac{13}{2}x + \frac{51}{4}x^2 + \frac{94}{6}x^3$ e) none of these

8. The series $\displaystyle\sum_{k=0}^{\infty} \frac{k^{2k}}{(2k)!}$

a) converges by the ratio test
b) diverges
c) converges by the root test
d) converges by the comparison test
e) none of these

9. The series $\displaystyle\sum_{k=20}^{\infty} \frac{1}{\sqrt{4k+8}}$

a) converges by the integral test
b) converges by the limit comparison test
c) diverges by the limit comparison test
c) diverges by the ratio test
d) none of these

10. $\displaystyle\lim_{x\to\infty}\left(1 + \frac{1}{x} + \frac{1}{x^2}\right)^x$

a) does not exist b) equals e
c) equals e^{-1} d) equals e^2 e)none of these

11. $\left\{\frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{n+6}\right\}_{n=1}^{\infty}$ is

a) an infinite series which converges
b) an infinite series which diverges
c) a sequence which does not converge to 0
d) a sequence which converges to 0
e) none of these

12. $\displaystyle\int_{-1}^{1} \frac{1}{x}\, dx$

a) equals 0 b) diverges c) equals $+2$ d) equals -2 e) none of these

13. $e^{\ln 7 + 6i} = a + bi$ where

a) = _____ and b) = _____

II SHOW ALL WORK.

1. Integrate:

   a) $\displaystyle\int_1^e \frac{\ln x\ dx}{\sqrt{x}}$ b) $\displaystyle\int \sqrt{16-x^2}\ dx$ c) $\displaystyle\int \frac{5x+3}{x^2-8x+16}\ dx$

2. Find the equation of the tangent line to the curve

   $x = t^3 - 4t + 1$
   $y = t^2 + 5$

   at the point where $t = 2$.

3. a) Find all values of z such that $z^4 = -1 + \sqrt{3}i$
   Express your answers in the form $r\text{cis}\theta$. [Note: $\text{cis}\theta = \cos\theta + i\sin\theta$]

   b) Plot and label the points in part (a) on a graph.

4. a) Let $f(x)$ be a function. For the Taylor series of $f(x)$ about a, write the general formula for $R_n(x)$, the Lagrange form of the remainder.

   b) Find the Taylor series for e^{2x} about $a = \ln 3$. Express your answer in $\sum$ notation.

   c) Find $R_5\left(\frac{1}{2} + \ln 3\right)$ for the particular series in part (b).

5. Let $f(x) = \displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{(3x)^k}{\ln k}$

   a) Determine the interval of convergence of this series. Show all work.

   b) Let $P_4(x)$ denote the Maclaurin polynomial for f whose highest power of x is x^4. Let u be the smallest upper bound for $\left|f\left(\frac{1}{6}\right) - P_4\left(\frac{1}{6}\right)\right|$, i.e., $\left|f\left(\frac{1}{6}\right) - P_4\left(\frac{1}{6}\right)\right| \le u$.
   Select the letter from the following list that correctly describes u. (Answer only will be graded.)

   a) $0 \le u \le \dfrac{1}{36\ln 5}$ b) $\dfrac{1}{36\ln 5} < u \le \dfrac{1}{30\ln 5}$ c) $\dfrac{1}{30\ln 5} < u \le \dfrac{1}{20\ln 5}$

   d) $\dfrac{1}{20\ln 5} < u \le \dfrac{1}{15\ln 5}$ e) $u > \dfrac{1}{15\ln 5}$

6. By integrating an appropriate series, find the Maclaurin Series for $\ln(1+4x)$. Express your answer in $\sum$ notation.