Final Review Spring 1990

1. Simplify

   a) $e^{3\ln x+2\ln 5}$ b) $\tan^{-1}\left(\tan\frac{4\pi}{3}\right)$ c) $\sin\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

2. Find x if $\ln(\frac{1}{x^2}) = e$

3. Find $\frac{dy}{dx}$ if

   a) $y = x(\sin^{-1} x)^2$ b) $y = x^{(x-e^{3x})}$

   c) $y = \dfrac{4x^2+5}{(x^2+1)^3\sqrt{x-3}}$ (use logarithmic differentiation)

4. Solve the differential equation $\frac{dy}{dx} = 2x(y^2+1)$ with $y(0) = -1$

5. Find $\left.\frac{dy}{dx}\right|_{t=1}$ and $\left.\frac{d^2y}{dx^2}\right|_{t=1}$ for

$$\begin{cases} x = t^7 + t^3 - 2 \\ y = 4t^2 + 5 \end{cases}$$

6. Integrate

   a) $\int_1^e \frac{\ln x}{\sqrt{x}}\,dx$ b) $\int_{-1}^2 \frac{1}{x}\,dx$ c) $\int \sin^4 x \cos^5 x\,dx$

   d) $\int \frac{x+1}{\sqrt{x^2+2x+5}}\,dx$ e) $\int \frac{2x^2-3x-1}{x^2-2x-3}\,dx$ f) $\int_3^7 \frac{dx}{x^2-6x+25}$

7.

   a) Sketch the curves $r = 1+\sin\theta$ and $r = 3\sin\theta$ on the same axes.

   b) Convert to rectangular coordinates $r^2 = \sin 2\theta$.

8.

   a) Use Simpson's Rule with $n = 4$ to approximate the integral $\int_0^{\frac{1}{2}} \frac{dx}{1+x^2}$. Express your answer as a sum. Do not simplify.

   b) Use Maclaurin Series to approximate the same integral so that the error in your approximation is less than $\frac{1}{100}$. Do not use a calculator.

9. Find

   a) $\lim_{x\to 0^-} \frac{\sin x + x}{x^2}$ b) $\lim_{x\to\infty} x^{\left(\frac{1}{x^2+1}\right)}$

10. Determine convergence or divergence

a) $\sum_{k=1}^{\infty} \sqrt{\frac{k-1}{k^3+6}}$ b) $\sum_{k=1}^{\infty} \frac{8^k}{2 \cdot 4 \cdot 6 \cdots (2k)}$ c) $\sum_{k=1}^{\infty} (-1)^k \frac{1}{(1+\frac{1}{k})^k}$

11. Find the interval of convergence of $\sum_{k=1}^{\infty} \frac{(-1)^k (2x+1)^k}{k+1}$

12.

a) Find the Maclaurin Series for $f(x) = \frac{1}{(1+x)^2}$ by finding the derivatives of $f^{(k)}(x)$.

b) Find the Taylor expansion for the same $f(x)$ about $a = 1$. Express both (a) and (b) in $\sum$ notation.

c) Find the Lagrange form of the remainder, $R_{15}(\pi)$, for the same $f(x)$ about $a = 1$.

13.

a) Find the 3rd. Taylor polynomial for $f(x) = \tan^{-1} x$ about $a = 1$.

b) Find the Lagrange form of the remainder, $R_2(\frac{3}{2})$ for $f(x) = \cos \pi x$ about $a = 1$.

14.

a) How many terms of the alternating harmonic series are needed to approximate $\ln 2$ to two decimal place accuracy? Do this without a calculator.

b) How many terms of the Maclaurin series for $\sin x$ must be used to compute $\sin(1)$ to within an accuracy of .0002? Do this without a calculator.

15. By integrating an appropriate series, find the Maclaurin series for $\ln(1+4x)$. Express your answer in $\sum$ notation.

16. Express in the form $x = iy$, x, y real:

a) $\frac{3+i}{1-2i}$ b) $3\text{cis}\frac{\pi}{6}$ c) $e^{2+\frac{\pi i}{3}}$ d) $(1+i)^{10}$

17. Find all complex sixth roots of -64. Leave your answer in the form $r\text{cis}\theta$. $[r\text{cis}\theta = r(\cos\theta + i\sin\theta)]$

18. Find all values of z such that $z^4 = -1 + \sqrt{3}i$. Plot and label the answers on a graph.

19. Compute the radius and center of the cricle of convergence of

$$\sum_{k=1}^{\infty} \frac{(z+3-4i)^k}{k^5 \cdot 5^k}$$

Sketch the circle in the complex plane.