Final Review Fall 1992

1. Determine whether the following series converge. Show your tests in detail. Specify the names of the tests you use. Give reasons for the convergence or divergence of any series you use in comparison or limit comparison tests.

   a) $\sum_{n=0}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{n! 3^n}$ b) $\sum \frac{(-1)^n n}{\sqrt{n^5+1}}$ c) $\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$

   d) $\sum_{n=1}^{\infty} (-1)^n \frac{3n+5}{2n-1}$ e) $\sum_{n=1}^{\infty} \frac{n^2-2}{n^6+5}$ f) $\sum_{n=0}^{\infty} \frac{(-1)^n (20)^n}{n!}$

   g) $\sum_{n=1}^{\infty} \sqrt{\frac{n^2+2}{n^4+2}}$ h) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ i) $\sum_{n=1}^{\infty} \left(\frac{n-1}{2n}\right)^n$

   j) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{\frac{3}{2}}}$ k) $\sum_{n=1}^{\infty} \frac{2^n}{n^2+1}$ l) $\sum_{n=1}^{\infty} \frac{2+\cos n}{n^2+1}$

2. For what values of x does the series (i) converge absolutely,and (ii) converge conditionally.

   a) $\sum_{n=0}^{\infty} \frac{(x-3)^n}{3^n n^2}$ b) $\sum_{n=1}^{\infty} \frac{3^n (3x+1)^n}{n}$

   c) $\sum_{n=0}^{\infty} \frac{3^n (x-2)^n}{n!}$ d) $\sum_{n=0}^{\infty} \frac{n!(2x-1)^n}{(1000)^n}$

3. State the definition of $f(x)$

   a) $f(x) = \ln x$ b) $f(x) = a^x, \quad a > 0$ c) $f(x) = \sec^{-1} x$

4. Simplify

   a) $e^{3 \ln x + 2 \ln 5}$ b) $\ln e^{3x}$ c) $\sin^{-1}\left(\sin \frac{4\pi}{3}\right)$ d) $\sin\left(\cos^{-1}\left(-\frac{3}{5}\right)\right)$

5. Find $\frac{dy}{dx}$

   a) $y = \ln(1-x)$ b) $y = x \cos^{-1}(3x)$ c) $y = (\sec^{-1} 2x)^3$

   d) $y = x^{(x-e^{3x})}$ e) $y = \frac{(x^2+5)(x-2)^3}{(x^2+1)\sqrt{x-3}}$

6. Suppose that the rate of change of the number y of bacteria in a culture is proportional to y. At noon, the number of bacteria present was 10^2. At 1pm it was $2(10^3)$. How many bacteria were there at 4 pm?

7. Find

a) $\displaystyle\int x^2 \sin x \; dx$ b) $\displaystyle\int \frac{e^x+1}{e^x+x}\; dx$ c) $\displaystyle\int_2^{\infty} \frac{1}{x\sqrt{\ln x}}\; dx$

d) $\displaystyle\int \sin^3 2x \cos^2 2x \; dx$ e) $\displaystyle\int \frac{x}{\sqrt{x^2+2x+5}}\; dx$ f) $\displaystyle\int e^x \sin 3x \; dx$

g) $\displaystyle\int \frac{2x^2-3x-1}{x^2-2x-3}\; dx$ h) $\displaystyle\int \sec^4 x \; dx$ i) $\displaystyle\int \tan^4 x \; dx$

j) $\displaystyle\int \frac{1}{x\sqrt{1-x^2}}\; dx$ k) $\displaystyle\int_4^3 \frac{1}{\sqrt{x^2-4}}\; dx$ l) $\displaystyle\int \frac{1}{(x^2+4)^2}\; dx$

8. Find $\displaystyle\lim_{n\to\infty} \left(\frac{n+2}{n+1}\right)^{3n}$

9. Find the sum $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{2^n} + \frac{2(-1)^n}{5^n}\right)$

10.

a) Find the Taylor series for $f(x) = e^{\frac{x}{3}}$ expanded about $a = 0$.
(Your answer should include a description of the nth term).

b) Use the first 5 terms of this Taylor series to obtain an approximation of $e^{\frac{1}{3}}$.
(You may leave your answer as a sum of unsimplified terms).

c) Show that the error in this approximation has magnitude less than 10^{-4}.

11. Sketch the curves whose polar equations are

a) $r = 3(1+\sin\theta)$ b) $r = 3\sin\theta$ c) $r = \sin^3\theta$

12. Convert to rectangular coordinates $r^2 = \sin\theta$

13. Identify and sketch. Label centers, vertices and asymptotes (if any).

a) $y^2 + 4x - 8 = 0$ b) $y^2 + 4y + x^2 - 4x = 0$ c) $(y-1)^2 - 4(x-2)^2 = 9$