Tufts University
Department of Mathematics

Math 12 - Fall 1993

Review for Final Exam

Reminder: The date of the Final Exam is Friday, December 17, 1993, at 3 p.m.

1. State the definition of $f(x)$

   a) $f(x) = \ln x$ b) $f(x) = a^x, \quad a > 0$ c) $f(x) = \sin^{-1} x$

2. Simplify

   a) $e^{3\ln x + 2\ln 5}$ b) $\ln e^{3x}$ c) $\sin^{-1}\left(\sin \frac{4\pi}{3}\right)$ d) $\sin\left(\cos^{-1}\left(-\frac{3}{5}\right)\right)$

3. Find $\dfrac{dy}{dx}$

   a) $y = \ln(1 - x)$ b) $y = x\cos^{-1}(3x)$ c) $y = (\tan^{-1} 2x)^3$

   d) $y = x^{(x - e^{3x})}$ e) $y = \dfrac{(x^2+5)(x-2)^3}{(x^2+1)\sqrt{x-3}}$

4. Suppose that the rate of change of the number y of bacteria in a culture is proportional to y. There are 100 present at noon, and 2000 present at 1 p.m. How many bacteria are present at 4 p.m.?

5. Find

   a) $\displaystyle\int x^2 \sin x \, dx$ b) $\displaystyle\int \frac{e^x+1}{e^x+x}\, dx$ c) $\displaystyle\int_2^\infty \frac{1}{x\sqrt{\ln x}}\, dx$

   d) $\displaystyle\int \frac{x}{\sqrt{x^2+2x+5}}\, dx$ e) $\displaystyle\int e^x \sin 3x \, dx$

   f) $\displaystyle\int \frac{2x^2-3x-1}{x^2-2x-3}\, dx$ g) $\displaystyle\int \frac{1}{x\sqrt{1-x^2}}\, dx$ h) $\displaystyle\int \frac{1}{(x^2+4)^2}\, dx$

6. Find $\displaystyle\lim_{n\to\infty}\left(\frac{n+2}{n+1}\right)^{3n}$

7. Given that the Maclaurin series for e^x is

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots,$$

   and that this series represents e^x for all values of x,

a) Find the Maclaurin series for e^{x^2}. For what values of x does this series represent e^{x^2}?

b) Use your answer from part (a) to find the Maclaurin series of the function

$$g(x) = \int_0^x e^{t^2}\, dt.$$

For what values of x does this series represent $g(x)$?

8. Given that the Maclaurin series for $\sin x$ is

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots,$$

and that this series represents $\sin x$ for all values of x,

a) Find the Maclaurin series for $\sin(3x)$. For what values of x does this series represent $\sin(3x)$?

b) Use your answer from part (a) to find the Maclaurin series of $\cos(3x)$. For what values of x does this series represent $\cos(3x)$?

9.

a) Find the fourth order Taylor Polynomial, $P_4(x)$, based at 1 for $f(x) = \ln x$.

b) Find a formula for the remainder, $R_4(x)$.

c) Find a good bound for $|R_4(x)|$ when $\frac{1}{2} \leq x \leq \frac{3}{2}$.

10. Sketch the graph of the given polar equations.

a) $r = 3(1 + \sin\theta)$ b) $r = 2\sin(3\theta)$

11. Find the Cartesian equation of the polar equation $r^2 = \sin\theta$.

12.

a) Use the Trapezoidal Rule with $n = 4$ to approximate the definite integral $\int_1^2 \frac{1}{x}\, dx$. Leave your answer as a sum.

b) Find a good bound for the error in part (a) by using the formula

$$E_n = -\frac{(b-a)^3}{12n^2} f''(c)$$

13. Estimate the positive root of $x^2 - 2 = 0$ by applying Newton's method twice, starting with $x_0 = 1$.

14. Determine whether the following series converge. For convergent series with both positive and negative terms, determine whether they converge absolutely or conditionally. Show your tests in detail. Specify the names of the tests you use. Give reasons for the convergence or divergence of any series you use in the comparison or limit comparison tests.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^2+1}$ b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{n^5+1}}$ c) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$

d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3n+5}{2n-1}$ e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-2}{n^6+5}$ f) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (20)^n}{n!}$

g) $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{n^2+2}{n^4+2}}$ h) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ i) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{\frac{3}{2}}}$

15. Find the convergence set of the given power series.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-3)^n}{3^n n^2}$ b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n (3x+1)^n}{n}$

c) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n (x-2)^n}{n!}$ d) $\displaystyle\sum_{n=0}^{\infty} \frac{n!(2x-1)^n}{(1000)^n}$

16. Determine whether the following series converge or diverge. Find the sums of those that converge.

a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5}{2^n} + \frac{2(-1)^n}{5^n}\right)$ b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$ c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4}{3}\right)^n$