Math 12 | Tufts University Department of Mathematics Review Problems for Final Exam | Fall, 1996

1) Determine which of the functions is one-to-one on the domain of definition D. To receive credit you must state clearly whether it is or is not one-to-one and your reasons.

(a) $f(x) = \dfrac{1-|x|}{x}, D = \{x | x \neq 0\}$.

(b) $f(x) = \tan x, D = \{x | -\pi/4 \leq x \leq \pi/4\}$.

(c) $f(x) = \sin x, D = \{x | -\pi \leq x \leq \pi\}$.

(d) $f(x) = \displaystyle\int_0^x \tan^{-1} t\, dt, D = \{x | -\infty < x < \infty\}$.

2) Find the derivatives:

(a) $\ln\left(\dfrac{a-x}{a+x}\right)$ (b) e^{-3x} (c) $\sin^{-1} 3x$ (d) $\tan^{-1} 3x$

3) If the half-life of Radium-226 is 1590 years, how long will it take for 10% of the radium to decay? [Hint: $\ln 2 \approx 0.69, \ln 3 \approx 1.10$, and $\ln 5 \approx 1.61$.]

4) Evaluate the limits:

(a) $\displaystyle\lim_{x\to\infty} x \ln \frac{x+5}{x-5}$ (b) $\displaystyle\lim_{x\to\pi} \frac{1}{x-\pi} + \frac{1}{\sin x}$

5) Find:

(a) $\displaystyle\int \frac{4x-7}{x^2-3x+2} dx$ (b) $\displaystyle\int \frac{-2x+6}{(x+1)(x^2+x+1)} dx$

(c) $\int \cos^2\theta \sin^3\theta\, d\theta$ (d) $\displaystyle\int \frac{x^3}{\sqrt{16-x^2}} dx$

6) Evaluate $\displaystyle\int_0^\infty e^{-x} \sin x\, dx$.

7) Sketch the graph of the parametric equation:

$$x = t^3 \quad y = t^2 \quad 0 \leq t \leq 4$$

8)

(a) Find the Cartesian coordinates corresponding to $(4, \pi/6)$ and polar coordinates corresponding to $(-3, \sqrt{3})$.

(b) Find the polar coordinates of the following points whose Cartesian coordinates are given

[1] $(-2\sqrt{3}, -2)$ [2] $(1, \sqrt{3})$

(c) Find the Cartesian coordinates of the following points

[1] $(4, \frac{\pi}{3})$ [2] $(7, \frac{-2\pi}{3})$

9) Determine whether the following converge or diverge:

(a) $a_n = \dfrac{n \cos n}{n^2 + 1}$ (b) $a_n = \dfrac{n}{\ln n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{1 + n^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{2n-1}{3n+4}}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(\ln n)^2}$ (f) $\displaystyle\sum_{n=1}^{\infty} \frac{3!3^n}{(n+1)!}$

10)

(a) The Maclaurin series for $\tan^{-1} x$ is

$$\tan^{-1} x = x - x^3/3 + x^5/5 - x^7/7 + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots \quad \text{for} \quad -1 < x < 1$$

(b) Use the series for $\tan^{-1} x$ to find the Maclaurin series for $\dfrac{1}{1 + x^4}$.

(c) Use the series for $\tan^{-1} x$ to express $\int_0^1 \tan^{-1}(\sqrt{x})$ as a series.

11) Find

(a) The real part, imaginary part, magnitude and complex conjugate of $-2 + 3i$.

(b) $\dfrac{1 + 2i}{3 + 4i}$

(c) $(1 + 2i)^2 - (3 + 4i) - 7$

(d) all roots of $x^2 - 2x + 17 = 0$

(e) $(1 - \sqrt{3}i)^7$

(f) all of the fourth roots of -25 and graph them

(g) $e^{2 - i\pi/4}, e^{1 + i2\pi/4}$ in the form $a + bi$.

For the next two problems 12) and 13), determine:

a) The Taylor series for the function about the given a.
b) Where the series converges. (Find the radius and interval of convergence.)
c) For which x the series converges absolutely; converges conditionally.
d) Whether the Taylor series about the given a equals the function.

12)

$$f(x) = \ln x, \quad a = 3$$

13)

$$f(x) = \frac{1}{(1-x)^2}, \quad a = 0$$

14) Determine the interval of convergence of the following series and specify any points at which the convergence is conditional:

a) $\displaystyle\sum_{n=0}^{\infty} \frac{nx^{n+1}}{5^n \ln n}$ b) $\displaystyle\sum_{n=0}^{\infty} \frac{n^n x^n}{n!}$ c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{3n}$

15) Find

a) $\dfrac{d}{dx} 5^{\ln x}$ b) $\dfrac{d}{dx} x^{\sin x}$ c) $e^{\ln 5x^3}$

16)

a) Find an interval on which $f(x) = \sin x^2$ is invertible.

b) Given $f(x) = x^2 - 2x + 1$ on $[1, 2]$ find a formula for the inverse of f and give the range of the inverse.

18) Find

a) $\log_7(49^2)$ b) $9^{\log_3 7}$ c) $e^{4\ln 2}/\log_5 625$