

- 1) If  $f(x) = \sin^{-1}(x^2 - 1)$ , find (a) the domain of  $f$ , (b)  $f'(x)$ , and (c) the domain of  $f'$   
 (d) If  $f(x) = 2x + \cos x$  and  $g = f^{-1}$ , find  $g'(1)$ .  
 (e) Simplify the expression  $\cos(\tan^{-1} x)$ .  
 (f) Evaluate  $\lim_{x \rightarrow 2^+} \arctan\left(\frac{1}{x-2}\right)$ .  
 (g) Differentiate  $y = \frac{1}{\sin^{-1} x}$  and (h) Differentiate  $f(x) = x \tan^{-1} \sqrt{x}$ .  
 (i) Find the roots of the equation  $x^2 + x + 1 = 0$ .  
 Write the following numbers in polar form: (j)  $z = 1 + i$  (k)  $w = \sqrt{3} - i$ .  
 (l) Find  $(\frac{1}{2} + \frac{1}{2}i)^{10}$ .  
 Evaluate (m)  $e^{i\pi}$  (n)  $e^{-1+i\pi/2}$ .  
 (o) Express the number  $\frac{-1+3i}{2+5i}$  in the form  $a + bi$ .
- 2) (a) By comparing areas, show that  $\frac{1}{2} < \ln 2 < \frac{3}{4}$ .  
 (b) Use the Midpoint Rule with  $n = 10$  to estimate the value of  $\ln 2$ .
- 3) (a) Expand the expression  $\ln \frac{(x^2+5)^4 \sin x}{x^3+1}$ .  
 (b) Express  $\ln a + \frac{1}{2} \ln b$  as a single logarithm.  
 (c) Find  $\frac{d}{dx} \ln \frac{x+1}{\sqrt{x-2}}$ . (d) Find  $f'(x)$  if  $f(x) = \ln |x|$ .  
 (e) Differentiate  $y = \frac{x^{3/4} \sqrt{x^2 + 1}}{(3x + 2)^5}$  (f) Solve the equation  $e^{5-3x} = 10$ .  
 (g) Find  $\lim_{x \rightarrow x} \frac{e^{2x}}{2^{2x} + 1}$ . (h) Differentiate the function  $y = e^{\tan x}$ .  
 (i) Find  $y'$  if  $y = e^{-4x} \sin 5x$ . (j) Differentiate  $y = x^{\sqrt{x}}$ .  
 Evaluate:  
 (k)  $\sin^{-1} \frac{1}{2}$  (l)  $\tan(\arcsin \frac{1}{3})$  (m)  $\sin(\sin^{-1} 0.6)$  (n)  $\sin^{-1}(\sin \frac{\pi}{12})$  (o)  $\sin^{-1}(\sin \frac{2\pi}{3})$
- 4) A bacteria culture starts with 1000 bacteria, and after 2 h the population is 2500 bacteria. Assuming that the culture grows at a rate proportional to its size, find the population after 6 h.
- 5) The *half-life* of radium-226 ( ${}^{226}_{88}\text{Ra}$ ) is 1590 years. This means that the rate of decay is proportional to the amount present, and half of any given quantity will disintegrate in 1590 years.  
 (a) A sample of radium-226 has a mass of 100 mg. Find a formula for the mass of ( ${}^{226}_{88}\text{Ra}$ ) that remains after  $t$  years.  
 (b) Find the mass after 1000 years correct to the nearest milligram.  
 (c) When will the mass be reduced to 30 mg?

- 6) Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings, provided that this difference is not too large. Suppose the object takes 40 min to cool from  $30^{\circ}\text{C}$  to  $24^{\circ}\text{C}$  in a room that is kept at  $20^{\circ}\text{C}$ .
- (a) What was the temperature of the object 15 min after it was  $30^{\circ}\text{C}$ ?
  - (b) How long will it take the object to cool down to  $21^{\circ}\text{C}$ ?