1. If $f(x) = \sin^{-1}(x^2 - 1)$, find (a) the domain of f, (b) $f'(x)$, and (c) the domain of f'
   (d) If $f(x) = 2x + \cos x$ and $g = f^{-1}$, find $g'(1)$.
   (e) Simplify the expression $\cos(\tan^{-1} x)$.
   (f) Evaluate $\displaystyle\lim_{x\to 2^+} \arctan\left(\frac{1}{x-2}\right)$.
   (g) Differentiate $y = 1/\sin^{-1} x$ (h) Differentiate $f(x) = x\tan^{-1}\sqrt{x}$.

2. (a) By comparing areas, show that $\frac{1}{2} < \ln 2 < \frac{3}{4}$.
   (b) Use the Midpoint Rule with $n = 10$ to estimate the value of $\ln 2$.

3. (a) Expand the expression $\displaystyle\ln\frac{(x^2+5)^4 \sin x}{x^3+1}$.
   (b) Express $\ln a + \frac{1}{2}\ln b$ as a single logarithm.
   (c) Find $\displaystyle\frac{d}{dx}\ln\frac{x+1}{\sqrt{x-2}}$. (d) Find $f'(x)$ if $f(x) = \ln|x|$.
   (e) Differentiate $\displaystyle y = \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$ (f) Solve the equation $e^{5-3x} = 10$.
   (g) Find $\displaystyle\lim_{x\to\infty}\frac{e^{2x}}{e^{2x}+1}$.

   Differentiate (h) $y = e^{\tan x}$ (i) $y = e^{-4x}\sin 5x$ (j) $y = x^{\sqrt{x}}$.
   Evaluate:
   (k) $\sin^{-1}\frac{1}{2}$ (l) $\tan(\arcsin\frac{1}{3})$ (m) $\sin(\sin^{-1} 0.6)$ (n) $\sin^{-1}(\sin\frac{\pi}{12})$ (o) $\sin^{-1}(\sin\frac{2\pi}{3})$

4. A bacteria culture starts with 1000 bacteria, and after 2 h the population is 2500 bacteria. Assuming that the culture grows at a rate proportional to its size, find the population after 6 h.

5. The *half-life* of radium-226 ($^{226}_{88}$Ra) is 1590 years. This means that the rate of decay is proportinal to the amount present, and half of any given quantity will disintegrate in 1590 years.

   (a) A sample of radium-226 has a mass of 100 mg. Find a formula for the mass of ($^{226}_{88}$Ra) that remains after t years.
   (b) Find the mass after 1000 years correct to the nearest milligram.
   (c) When will the mass be reduced to 30 mg?

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6.

(a) If \$500 is borrowed at 14% interest, find the amounts due at the end of 2 years if the interest is compounded (a) annually, (b) quarterly, (c) monthly, (d) daily, (e) hourly, and (f) continuously.

(b) How long will it take an investment to double in value if the interest rate is 6% compounded continuously?

7. Find (a) $\lim_{x\to 0} \frac{2^x - 1}{x}$ (b) $\lim_{x\to\infty} \frac{e^x}{x^2}$ (c) $\lim_{x\to\infty} \frac{\ln x}{\sqrt[3]{x}}$ (d) $\lim_{x\to 0} \frac{\tan x - x}{x^3}$

(e) $\lim_{x\to\pi^-} \frac{\sin x}{1-\cos x}$ (f) $\lim_{x\to 0+} x\ln x.$ (g) $\lim_{x\to(\pi/2)^-} (\sec x - \tan x)$

(h) $\lim_{x\to 0+} (1+\sin 4x)^{\cot x}$

8. Find (a) $\int x^2 e^{x^3}\, dx$ (b) $\int \frac{x}{x^2+1}\, dx$ (c) $\int \tan x\, dx$ (d) $\int \frac{1}{\sqrt{1-4x^2}}\, dx$

(e) $\int \frac{1}{x^2+a^2}\, dx$ (f) $\int \frac{x}{x^4+9}\, dx$ (g) $\int x\sin x\, dx$ (h) $\int \ln x\, dx$

(i) $\int x^2 e^x\, dx$ (j) $\int e^x \sin x\, dx$ (k) $\int \cos^3 x\, dx$ (l) $\int \sin^5 x\cos^2 x\, dx$

(m) $\int \sin 4x\cos 5x\, dx$ (n) $\int \frac{\sqrt{9-x^2}}{x^2}\, dx$ (o) $\int \frac{1}{x^2\sqrt{x^2+4}}\, dx$

(p) $\int \frac{x}{\sqrt{x^2+4}}\, dx$ (q) $\int \frac{x}{\sqrt{3-2x-x^2}}\, dx$ (r) $\int \frac{x^3+x}{x-1}\, dx$

(s) $\int \frac{x^2+2x-1}{2x^3+3x^2-2x}\, dx$ (t) $\int \frac{dx}{x^2-a^2}$, where $a \neq 0$.

(u) $\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}\, dx$ (v) $\int \frac{2x^2-x+4}{x^3+4x}\, dx$

(w) $\int \frac{4x^2-3x+2}{4x^2-4x+3}\, dx$ (x) $\int \frac{1-3x+2x^2-x^3}{x(x^2+1)^2}\, dx$

(y) $\int \frac{\sqrt{x+4}}{x}\, dx$ (z) $\int \frac{dx}{\sqrt{x}-\sqrt[3]{x}}$

(aa) the form of the partial fraction decomposition of $\frac{x^3+x^2+1}{x(x-1)(x^2+x+1)(x^2+1)^3}$

(bb) $\int \frac{x^2}{\sqrt{5-4x^2}}dx$ using a table of integrals

(cc) the values of p for which the integral $\int_1^\infty \frac{1}{x^p}\, dx$ is convergent

9. Find (a) the area under the curve $y = e^{-3x}$ from 0 to 1. (b) $\displaystyle\int_1^e \frac{\ln x}{x}\,dx$

(c) $\displaystyle\int_0^1 \tan^{-1} x\,dx$ (d) the area enclosed by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

(e) $\displaystyle\int_0^{3\sqrt{3/2}} \frac{x^3}{(4x^2+9)^{3/2}}\,dx$ (f) $\displaystyle\int_0^{\pi/2} \sec x\,dx$ (g) $\displaystyle\int_0^3 \frac{dx}{x-1}$

10. Use (a) the Trapezoidal Rule with $n = 5$ (b) the Midpoint Rule with $n = 5$
(c) Simpson's Rule with $n = 10$ to approximate $\int_1^2 (1/x)dx$.

11. Solve the initial value problem
(a) $\dfrac{dy}{dx} = \dfrac{6x^2}{2y + \cos y}$, $y(1) = \pi$ (b) $y' = 1 + y^2 - 2x - 2xy^2$, $y(0) = 0$

12. Find (a) $\displaystyle\lim_{n\to\infty} \frac{n}{n+1}$ (b) $\displaystyle\lim_{n\to\infty} \frac{\ln n}{n}$ (c) $\displaystyle\lim_{n\to\infty} (-1)^n$ (d) $\displaystyle\lim_{n\to\infty} \frac{(-1)^n}{n}$
(e) the values of r for which the sequence $\{r^n\}$ is convergent.

13. (a) Show that the sequence $a_n = n/(n^2+1)$ is decreasing
(b) Write the number $2.3\overline{17} = 2.3171717\cdots$ as a ratio of integers.

14. Find the sum of the series or show that it diverges.
(a) $\displaystyle\sum_{n=1}^{\infty} 2^{2n} 3^{1-n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{n(n+1)} + \frac{1}{2^n}\right)$ (e) $\displaystyle\sum_{n=0}^{\infty} x^n$, where $|x| < 1$.

15. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.
(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+1}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^7}}$ (f) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ (g) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

16. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ correct to three decimal places.

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17. Find the radius of convergence and interval of convergence of the series

(a) $\sum_{n=0}^{\infty} n!x^n$ (b) $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$ (c) $\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$

(d) $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$ (e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$

18. Find a power series representation of

(a) $x^3/(x+2)$ (b) $1/(1-x)^2$ (c) $\tan^{-1} x$ (d) $\int [1/(1+x^7)]\, dx$.

(e) Use part (d) to approximate $\int_0^{0.5} [(1/(1+x^7)]\, dx$ correct to within 10^{-7}.

19. (a) Find the Taylor series for $f(x) = e^x$ at $a = 2$

(b) Find the Maclaurin series for the function $f(x) = x \cos x$

(c) Represent $f(x) = \sin x$ as sum of its Taylor series centered at $\pi/3$.

(d) Evaluate $\int e^{-x^2} dx$ as an infinite series.

(e) Evaluate $\int_0^1 e^{-x^2} dx$ correct to within an error of 0.001

(f) Approximate the function $f(x) = \sqrt[3]{x}$ by a Taylor polynomial of degree 2 at $a = 8$.

(g) How accurate is this approximation when $7 \leq x \leq 9$?

(h) What is the maximum error possible in using the approximation

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

when $-0.3 \leq x \leq 0.3$? Use the approximation to find $\sin 12°$ correct to six decimal places.

(i) For what values of x is this approximation accurate to within 0.00005?

20. State the definition of each of the functions below and give their domains.

(a) $f(x) = \ln x$

(b) $f(x) = a^x$. (In addition to stating the domain of f, for which values of a does your definition make sense?)

(c) $f(x) = \cos^{-1} x$