

Final Review

**Supplementary Review Problems on Complex Numbers**

1. Evaluate (i.e., write in the form  $x + iy$ ,  $x$  &  $y$  real):

a)  $z = \frac{1+i}{1-i}$       b)  $x = e^{(1-i)}$       c) all  $z$  such that  $z^3 = 2i - 2$

2. Find each limit, if it exists:

a)  $\lim_{z \rightarrow i} \frac{z^2 + 2}{z^2 + 3}$       b)  $\lim_{z \rightarrow i} \frac{z^2 + 1}{z - i}$       c)  $\lim_{z \rightarrow 1+i} \frac{z^2 - 2iz - 2}{z - 1 - i}$

3. Find the radius of convergence:

a)  $\sum_{k=1}^{\infty} \frac{z^k}{k^2 3^k}$       b)  $\sum_{k=0}^{\infty} \frac{z^k}{k^3}$       c)  $\sum_{k=0}^{\infty} \frac{z^k}{k^k}$

4. Show that the series  $\sum_{k=0}^{\infty} z^k$  diverges at  $z = i$ .