4/7/2009

#1, page 321, Chapter 3 Review

$$(S_N)\quad \begin{aligned} x_1' &= 5x_1 - 2x_2 + 4e^{3t} \qquad & x_1(0) &= -1 \\ x_2' &= 4x_1 + x_2 + 4e^{3t} \qquad & x_2(0) &= 6 \end{aligned}$$

$$A = \begin{bmatrix} 5 & -2 \\ 4 & 1 \end{bmatrix} \qquad p(\lambda) = \begin{vmatrix} 5-\lambda & -2 \\ 4 & 1-\lambda \end{vmatrix}$$

$$= (\lambda-5)(\lambda-1) + 8$$

$$\lambda = \frac{6 \pm \sqrt{36-52}}{2} \qquad = \lambda^2 - 6\lambda + 13$$

$$= 3 \pm 2i$$

$$\underline{\lambda = 3-2i} \quad \left[\begin{array}{cc|c} 2+2i & -2 & 0 \\ 4 & -2+2i & 0 \end{array}\right] \xrightarrow{(2-2i)\times ①} \left[\begin{array}{cc|c} 8 & -4+4i & 0 \\ 4 & -2+2i & 0 \end{array}\right]$$

$$\xrightarrow[②-\frac{1}{2}①]{\frac{1}{8}\times ①} \left[\begin{array}{cc|c} 1 & -\frac{1}{2}+\frac{1}{2}i & 0 \\ 0 & 0 & 0 \end{array}\right] \quad \text{Use } \vec{v} = \begin{bmatrix} 1 \\ 1+i \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1+i \end{bmatrix} e^{(3-2i)t} = \begin{bmatrix} 1 \\ 1+i \end{bmatrix} e^{3t} (\cos 2t - i \sin 2t)$$

$$= \begin{bmatrix} \cos 2t \\ \cos 2t + \sin 2t \end{bmatrix} e^{3t} + i \begin{bmatrix} -\sin 2t \\ \cos 2t - \sin 2t \end{bmatrix} e^{3t}$$

$$\boxed{\vec{h}_1(t) = \begin{bmatrix} \cos 2t \\ \cos 2t + \sin 2t \end{bmatrix} e^{3t} \quad \vec{h}_2(t) = \begin{bmatrix} -\sin 2t \\ \cos 2t - \sin 2t \end{bmatrix} e^{3t}}$$

#1, p 321

Substitute $\vec{p}(t) = c_1(t)\vec{h}_1(t) + c_2(t)\vec{h}_2(t)$ into (S_N).

Solve

$$\left[\begin{array}{cc|c} \cos 2t & -\sin 2t & 4 \\ \cos 2t + \sin 2t & \cos 2t - \sin 2t & 4 \end{array}\right]$$

$$\xrightarrow[②-①]{} \left[\begin{array}{cc|c} \cos 2t & -\sin 2t & 4 \\ \sin 2t & \cos 2t & 0 \end{array}\right]$$

$$\xrightarrow[\sin 2t \times ②]{\cos 2t \times ①} \left[\begin{array}{cc|c} \cos^2 2t & -\cos 2t \sin 2t & 4\cos 2t \\ \sin^2 2t & \cos 2t \sin 2t & 0 \end{array}\right]$$

$$\xrightarrow[② \div \sin^2 2t]{①+②} \left[\begin{array}{cc|c} 1 & 0 & 4\cos 2t \\ 1 & \cot 2t & 0 \end{array}\right]$$

$$\xrightarrow[\substack{②-① \text{ and} \\ \tan 2t * ②}]{} \left[\begin{array}{cc|c} 1 & 0 & 4\cos 2t \\ 0 & 1 & -4\sin 2t \end{array}\right] \qquad \begin{array}{l} c_1(t) = 2\sin 2t \\ c_2(t) = 2\cos 2t \end{array}$$

$$\vec{p}(t) = 2\begin{bmatrix} \cos 2t \sin 2t \\ \cos 2t \sin 2t + \sin^2 2t \end{bmatrix} e^{3t} + 2\begin{bmatrix} -\cos 2t \sin 2t \\ \cos^2 2t - \cos 2t \sin 2t \end{bmatrix} e^{3t}$$

$$\boxed{\vec{p}(t) = \begin{bmatrix} 0 \\ 2 \end{bmatrix} e^{3t}}$$

(a) $\boxed{\vec{x}(t) = c_1\vec{h}_1(t) + c_2\vec{h}_2(t) + \vec{p}(t)}$

Check: $\vec{h}_1'(t) = \begin{bmatrix} 3\cos 2t - 2\sin 2t \\ 3\cos 2t + 3\sin 2t - 2\sin 2t + 2\cos 2t \end{bmatrix} e^{3t}$

$$= \begin{bmatrix} 3\cos 2t - 2\sin 2t \\ 5\cos 2t + \sin 2t \end{bmatrix} e^{3t}$$

$$A\vec{h}_1(t) = \begin{bmatrix} 5\cos 2t - 2\cos 2t - 2\sin 2t \\ 4\cos 2t + \cos 2t + \sin 2t \end{bmatrix} e^{3t} \quad \checkmark$$

$$h_2'(t) = \begin{bmatrix} -3\sin 2t - 2\cos 2t \\ 3\cos 2t - 3\sin 2t - 2\sin 2t - 2\cos 2t \end{bmatrix} e^{3t}$$

$$= \begin{bmatrix} -3\sin 2t - 2\cos 2t \\ \cos 2t - 5\sin 2t \end{bmatrix} e^{3t}$$

$$A\vec{h}_2(t) = \begin{bmatrix} -5\sin 2t - 2\cos 2t + 2\sin 2t \\ -4\sin 2t + \cos 2t - \sin 2t \end{bmatrix} e^{3t} \quad \checkmark$$

$$D\vec{x} - A\vec{x} = D\vec{p}(t) - A\vec{p}(t)$$

$$= \begin{bmatrix} 0 \\ 6 \end{bmatrix} e^{3t} - \begin{bmatrix} -4 \\ 2 \end{bmatrix} e^{3t} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} e^{3t} \quad \checkmark$$

$$\vec{x}(0) = c_1\vec{h}_1(0) + c_2\vec{h}_2(0) + \vec{p}(0) \overset{\text{set}}{=} \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$c_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \end{bmatrix} \qquad \begin{matrix} c_1 = -1 \\ c_2 = 5 \end{matrix}$$

(b) $\boxed{\vec{x}(t) = -\vec{h}_1(t) + 5\vec{h}_2(t) + p(t)}$