Mathematics 38 **Exam I**
Differential Equations **October 1, 2007**

No calculators, notes, or books are allowed. Please make sure all electronic devices you carry are turned off and put away out of sight.

Remember to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on this exam. Good luck!

1. (3 points each, no partial credit) For each of the differential equations below determine the order, determine whether the differential equation is linear, and if so, whether it is homogeneous.

a. $t^4\dfrac{d^3x}{dt^3} + t\dfrac{dx}{dt} - x - t^7 = 0$

b. $x^8\dfrac{dx}{dt} + \dfrac{d^7x}{dt^7} = x + t^9$

c. $(\dfrac{dx}{dt})^5 + \dfrac{d^4x}{dt^4} - t^3x^7 + t^7 = 0$

d. $(x')^2x''' = x^4x'' + t^5x'$

2. (3 points each, no partial credit) Find all real values of α for which the given function is a solution of the given differential equation.

a. $x = \alpha, \qquad \dfrac{d^7x}{dt^7} + \dfrac{dx}{dt} - x = 7$

b. $x = t^\alpha, t > 0, \qquad 16t^2xx'' + 3x^2 = 0$

c. $x = e^{\alpha t}, \qquad x'\sqrt{x} = 2e^{3t}$

3. (1 point each) For each of the following differential equations state whether it is normal on $0 < t < 2$.

a. $(t-1)\dfrac{dx}{dt} - 5x = 3t$

b. $3\dfrac{dx}{dt} - 5x = \csc \pi t$

c. $t\dfrac{dx}{dt} + e^tx = \sin t$

d. $t\sin t\dfrac{dx}{dt} + \pi x = \ln t$

4. (10 points) At the point $(t_0, \alpha) = (1, 2)$ the Existence and Uniqueness Theorem does not apply to the differential equation

$$(t-1)\frac{dx}{dt} = -x.$$

Determine whether there are no solutions, more than one solution or a unique solution of the differential equation with the initial value $x(1) = 2$. Explain!

5. (10 points) Determine whether the functions t^3 and $|t^3|$ are linearly independent on $-\infty < t < \infty$. Explain!
(Hint: Computing the Wronskian may *not* be the best approach.)

6. (10 points) Find the general solution of $x^{(5)} - 2x^{(4)} + x^{(3)} = 0$.

7. (15 points) Solve the initial-value problem $16x'' + x = 0$ with $x(0) = 2$ and $x'(0) = 9$.

8. (15 points) Find the general solution of $(D-1)^4x = 2t + e^{-t}$.

9. (15 points) Find the general solution of $x'' + x = \sec t, \qquad 0 < t < \pi/2$.

END OF EXAMINATION