Mathematics 38 — Tufts University — **Differential Equations**

Examination 1 — **October 5, 2009**

No calculators, notes, or books are allowed. Please make sure all electronic devices are turned off and out of sight.
Show all work and cross out work you do not want graded!
Remember to sign your blue book.
With your signature you are pledging that you have neither given nor received assistance on this exam. Good luck!

1. (5 points, answer only, no partial credit) Can the system below be solved by Cramer's rule?

$$\begin{aligned} x_1+\ x_2+x_3 &= 0\\ 2x_1+3x_2+x_3 &= 0\\ -x_1-3x_2+x_3 &= 0 \end{aligned}$$

2. (5 points, answer only, no partial credit) Consider the following system of equations:

$$\begin{aligned} x_1+\ x_2+x_3 &= 0\\ 2x_1+3x_2+x_3 &= 0\\ -x_1-3x_2+x_3 &= 0 \end{aligned}$$

Choose one answer: The system has
a. a unique solution, **b.** no solution, **c.** more than 1 solution. **d.** None of the above.

3. (5 points, answer only, no partial credit) Choose one answer.

$$\det\begin{pmatrix} 5 & 1 & \sin t & t^2+3 & 1\\ 0 & 4 & e^t & e^t & 0\\ 0 & 0 & 3 & \ln t & 8\\ 0 & 0 & 0 & 2 & \sqrt{t}\\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} =$$

a. 5, **b.** $2\sqrt{t}$, **c.** 120, **d.** $120-\sin t-\frac{1}{4}e^t-\frac{1}{4}(t^2+3)e^t$. **e.** None of the above.

4. (5 points, answer only, no partial credit) Choose one answer.
The functions e^t, e^{-t}, e^{t+1} are
a. linearly dependent **b.** linearly independent. **c.** None of the above.

5. (5 points, answer only, no partial credit) Consider the differential equation

$$t^4x''-2xx'=0.$$

Choose *every* function from the following that is a solution of this differential equation.
a. 0, **b.** 1, **c.** t, **d.** t^2, **e.** t^3, **f.** t^4, **g.** None of the above.

6. (5 points, answer only, no partial credit) Choose the correct answer:

$$(D-1)^3(t^4e^t)=$$

a. $(4t^3+4)e^t$, **b.** $4t^3e^t$, **c.** $(4t^3+24t^2+24t)e^t$, **d.** $(-t^4+8t^3+12t^2-24t)e^t$, **e.** $12t^2e^t$, **f.** $24te^t$,
g. None of the above.

7. (4 points, answer only, no partial credit) Consider the differential equation

$$(t-1)x'=-x.$$

Choose all correct answers from below. For the initial value $(t_0,\alpha)=(1,0)$:

- **a.** The theorem about existence and uniqueness of solutions applies, and the differential equation has a unique solution with this initial value.
- **b.** The theorem about existence and uniqueness of solutions applies, and the differential equation has no solution with this initial value.
- **c.** The theorem about existence and uniqueness of solutions applies, and the differential equation has more than one solution with this initial value.
- **d.** The theorem about existence and uniqueness of solutions does not apply, and the differential equation has a unique solution with this initial value.
- **e.** The theorem about existence and uniqueness of solutions does not apply, and the differential equation has no solution with this initial value.
- **f.** The theorem about existence and uniqueness of solutions does not apply, and the differential equation has more than one solution with this initial value.
- **g.** None of the above.

Examination continues on next page

8. (4 points, answer only, no partial credit) Again consider the differential equation $(t-1)x' = -x$ but now with the initial value $(t_0, \alpha) = (0, 1)$. From the statements **a.–g.** in the previous problem choose all that are correct for this new initial value.

9. (4 points, answer only, no partial credit) Yet again consider the differential equation $(t-1)x' = -x$ but now with the initial value $(t_0, \alpha) = (1, 1)$. From the statements **a.–g.** in Problem 7. choose all that are correct for this initial value.

10. (10 points, answer only, no partial credit) Consider the differential equation

$$(D-1)^2 x = e^t + t.$$

If one uses the method of undetermined coefficients, then the correct *simplified guess* for a particular solution is

a. $A + Bt^2e^t$, **b.** $A + Bt + Ce^t$, **c.** $A + Bt + Cte^t$, **d.** $Ae^t + Bte^t$, **e.** $At + Ce^t$,
f. $A + Bt + Ct^2e^t$, **g.** $At + B\sin t + C\cos t$, **h.** $A + Bt + C\sin t + D\cos t$, **i.** $A + Bt^2e^t + C\sin t + D\cos t$,
j. $A + Bt + Ce^t + D\sin t + E\cos t$, **k.** $A + Bt + Cte^t + D\sin t + E\cos t$, **l.** $At + Be^t + C\sin t + D\cos t$,
m. $A + Bt + Ct^2e^t + D\sin t + E\cos t$, **n.** None of the above.

11. (15 points) Solve the initial-value problem $\dfrac{dx}{dt} - tx = t, \quad x(0) = \dfrac{1}{2}$.

12. (10 points) Find the general solution of $\dfrac{d^4x}{dt^4} - x = 0$.

13. (5 points) Determine constants k_1 and k_2 such that $p(t) = k_1 t\cos 2t + k_2 t\sin 2t$ is a solution of

$$(D^2 + 4)x = \sin 2t.$$

14. (5 points) Solve the following system of equations *for x only*:

$$\begin{aligned} x + 2y - 3z &= 1 \\ 2x + 2y + 3z &= 2 \\ x - y + z &= 0 \end{aligned}$$

15. (7 points) Suppose $f(t)$ is a continuous function. Give a solution of the initial-value problem $x' + f(t)x = 0$, $x(1) = 0$.
(Hint: Think before applying standard techniques.)

END OF EXAMINATION