

No calculators, notes, books, pagers, mobile phones or other electronic devices are allowed on the exam. All answers should be in terms of real numbers and functions. You must **show all your work** to receive credit. *You are required to sign your exam book. With your signature, you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

1. (3 points each, no partial credit)

(a) Is $\frac{d^4x}{dt^4} + 5t^3 \frac{dx}{dt} = \sqrt{t^2 - 1}$ linear?

(b) Is the linear o.d.e. $t \frac{d^2x}{dt^2} - e^t \frac{dx}{dt} + tx = \sin t$ normal on the interval $(0, 2)$?

(c) Determine the largest intervals on which $5 \frac{d^2x}{dt^2} - 3 \frac{dx}{dt} = t$ is normal.

(d) Compute $(D^2 + 3D + 2)t^2e^{-t}$.

2. (10 points) Let $f(t, x) = \frac{-x}{(t-2)}$

(a) Is $f(t, x)$ continuous at $(2, 1)$?

(b) Is $\frac{\partial f}{\partial x}$ continuous at $(2, 1)$?

(c) Does the equation $\frac{dx}{dt} = \frac{-x}{t-2}$ $x(2) = 1$ satisfy the hypotheses of the existence and uniqueness theorem?

(d) Solve $\frac{dx}{dt} = \frac{-x}{t-2}$. How many solutions satisfy $x(2) = 1$?

(e) Why this does not violate the existence and uniqueness theorem?

3. (18 points) Find all real numbers α such that

(a) t^α is a solution of $(t^2D^2 - 3tD + 2)x = 0$.

(b) $2\alpha + 2\alpha t + \alpha t^2$ is a solution of $(D - 1)x = t^2$.

Exam Continues on Other Side

4. (20 points) Solve $x' - x = e^t \sin t$, $x(0) = 1$.

5. (20 points)

(a) Compute $\det \begin{bmatrix} e^t & \sin t & \cos t \\ e^t & \cos t & -\sin t \\ e^t & -\sin t & -\cos t \end{bmatrix}$

(b) Use a) to show that $c_1 e^t + c_2 \sin t + c_3 \cos t$ is the general solution of $(D - 1)(D^2 + 1)x = 0$

6. (20 points)

(a) Find the general solution of $(D^3 - 2D^2 + D)x = 0$.

(b) Find the solution for which $x(0) = x'(0) = 0$, $x''(0) = 1$.

End of Exam