Tufts
UNIVERSITY

Mathematics 38 | **Differential Equations**
Examination 1 | **February 18, 2010**

No calculators, notes, or books are allowed. Please make sure all electronic devices are turned off and out of sight.
Show all work and cross out work you do not want graded!
Remember to sign your blue book.
With your signature you are pledging that you have neither given nor received assistance on this exam. Good luck!
Please put the answers to problems 1–10 on the blue book cover in the corresponding box, as shown here:

1	Yes
2	w.
3	e.
4	c.
5	a.n.
6	cos(t)
7	high
8	yes
9	
10	
T	

1. (5 points, answer only, no partial credit) Can the system below be solved by Cramer's rule?

$$\begin{aligned} x+2y+\ z &= 0 \\ 3x+6y+3z &= 0 \\ -x\quad\ +\ z &= 0 \end{aligned}$$

2. (5 points, answer only, no partial credit) Consider the following system of equations:

$$\begin{aligned} x+2y+\ z &= 0 \\ 2x-\ y+3z &= 0 \\ 2x-6y+4z &= 0 \end{aligned}$$

Choose one answer: The system has
a. a unique solution, **b.** no solution, **c.** more than 1 solution. **d.** None of the above.

3. (5 points, answer only, no partial credit) Choose one answer.

$$\det\begin{pmatrix} t & 1 & \sin t & \ln t & 1 \\ t & 4 & \cos t & e^t & 3 \\ 0 & 0 & \tan t & t & \sqrt{t} \\ 0 & 0 & 0 & t^2 & 9 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} =$$

a. $4t^3\tan t$, **b.** $t\sin t\ln t$, **c.** $3t^3\tan t - 3t^2$, **d.** None of the above.

4. (5 points, answer only, no partial credit) Consider the differential equation

$$(t-3)\frac{dx}{dt} = 2x.$$

Choose all correct answers from below. For the initial value $(t_0, \alpha) = (3, 1)$:

a. The theorem about existence and uniqueness of solutions applies, and the differential equation has a unique solution with this initial value.
b. The theorem about existence and uniqueness of solutions applies, and the differential equation has no solution with this initial value.
c. The theorem about existence and uniqueness of solutions applies, and the differential equation has more than one solution with this initial value.
d. The theorem about existence and uniqueness of solutions does not apply, and the differential equation has a unique solution with this initial value.
e. The theorem about existence and uniqueness of solutions does not apply, and the differential equation has no solution with this initial value.
f. The theorem about existence and uniqueness of solutions does not apply, and the differential equation has more than one solution with this initial value.
g. None of the above.

5. (5 points, answer only, no partial credit) Again consider the differential equation $(t-3)\dfrac{dx}{dt} = 2x$ but now with the initial value $(t_0, \alpha) = (0, 1)$. From the statements **a.–g.** in Problem 4 choose all that are correct for these new initial data.

Examination continues on next page

6. (5 points, answer only, no partial credit) Yet again consider the differential equation $(t-3)\dfrac{dx}{dt} = 2x$ but now with the initial value $(t_0, \alpha) = (3, 0)$. From the statements **a.–g.** in Problem 4 choose all that are correct for these initial data.

7. (5 points, answer only, no partial credit) Compute

$$(D+3)^4(t^5 e^{-3t})$$

8. (10 points, answer only, no partial credit) Consider the differential equation

$$D(D-1)^2 x = 1 + te^t.$$

If one uses the method of undetermined coefficients, then the correct *simplified guess* for a particular solution is
a. $A + Bt^2e^t$, **b.** $A + Bt + Ce^t$, **c.** $A + Bt + Cte^t$, **d.** $Ae^t + Bte^t$, **e.** $At + Ce^t$, **f.** $A + Bt + Ct^2e^t$,
g. $A + Bt + Cte^t + Dt^2e^t$, **h.** $At + Bt^2e^t + Ct^3e^t$, **i.** $At + Bte^t + Ct^2e^t$, **j.** $At + B\sin t + C\cos t$,
k. None of the above.

9. (5 points, answer only, no partial credit) Choose one answer.
The functions e^{3t}, t, e^{-2t}, e^{3t+1} are
a. linearly dependent **b.** linearly independent. **c.** None of the above.

10. (5 points, answer only, no partial credit) Consider the differential equation

$$t^2x'' + tx' - x = 0.$$

Choose *every* function from the following that is a solution of this differential equation.
a. 0, **b.** 1, **c.** t, **d.** $1/t$, **e.** t^2, **f.** t^3, **g.** $\dfrac{t^2+1}{t}$, **h.** e^t, **i.** e^{-t}, **j.** $3te^t\sin t$,
k. None of the above.

11. (12 points) Find the general solution of $(D^2+1)^4x = 0$.

12. (20 points) Solve the initial-value problem

$$(D^2+1)x = \sin 3t, \qquad x(0) = 0, \quad x'(0) = 1.$$

13. (8 points) Solve the following system of equations *for x only*:

$$\begin{aligned} x + y + z &= 1 \\ 3x + 2y - z &= 0 \\ x - y + z &= 0 \end{aligned}$$

14. (5 points) Demonstrate that the system

$$\begin{aligned} 3x_1 + x_2 - x_3 + 4x_4 + 7x_5 &= 0 \\ x_1 + x_2 + x_3 - x_4 + x_5 &= 0 \\ x_1 - x_3 + x_5 &= 0 \\ 2x_1 + x_2 + x_4 + 2x_5 &= 0 \\ x_1 + 3x_2 + x_3 - x_4 + 5x_5 &= 0 \end{aligned}$$

has a solution.

END OF EXAMINATION