These solutions have not been proofread - they may be rife with typographical errors!

1. a) We need $t \neq 0$ and in the initial condition we have $t = 1$, so the largest rectangle is given by the condition $t > 0$.
b) Separate variables: $1/x = \int x'/x\,dx = \int 1/t\,dt = (1/t) + C$, and inserting $t = 1$, $x = 2$ gives $1/2 = 1/1 + C$, so $C = -1/2$ and $x(t) = \dfrac{1}{(1/t) - (1/2)}$. (Check by plugging in!)
c) The largest interval (including 1) on which this is defined is $(0, 2)$. (However, we could extend the solution by writing it as $\dfrac{t}{1 - (t/2)}$, which is defined on $(-\infty, 2)$.)

2. Solve the associated homogeneous equation by separation of variables:
$\ln|x| = \int x'/x\,dx = -\int 2t/(t^2+1)\,dt = -\ln(t^2+1) + C$, so $x(t) = \dfrac{k}{t^2+1}$ is the general solution of the associated homogeneous equation. Now try a solution $x(t) = \dfrac{k(t)}{t^2+1}$ to get $k'(t) = \dfrac{\cos t}{t^2+1}(t^2+1) = \cos t$, so $k(t) = \sin t + C$ and $x(t) = \dfrac{C + \sin t}{t^2+1}$. Inserting $t = 0$ and $x = 2$ gives $C = 2$, so the desired solution is $x(t) = \dfrac{2 + \sin t}{t^2+1}$.

3. a) The equilibria are $-3/2$, $1/2$, 4. The phase portrait shows that $-3/2$ is an attractor, 4 is a repeller, and $1/2$ is neither an attractor nor a repeller. $-3/2$ is stable, and $1/2$ and 4 are unstable equilibria.
b) The phase portrait is (excuse the gaps): $\rightarrow - \bullet - \leftarrow - \bullet - \leftarrow - \bullet \rightarrow -$
c) The phase portrait shows that a solution with $1/2 < x(0) < 4$ satisfies $\lim_{t\to\infty} x(t) = 1/2$ and $\lim_{t\to-\infty} x(t) = 4$, i.e., it converges to $1/2$ as $t \to \infty$ and to 4 as $t \to -\infty$.

4. a) The largest interval containing $t = 1$ on which $(t^2D^2 - 5tD + 5)x = 0$ is normal is $(0, \infty)$, i.e., it is given by the condition $t > 0$.
b) Plugging in successively we get the following:
t: $0 - 5t \cdot 1 + 5t = 0 \quad \checkmark$,
t^3: $t^2 \cdot 6t - 5t \cdot 3t^2 + 5t^3 = 6t^3 \neq 0 \quad \times$,
t^5: $t^2 \cdot 20t^3 - 5t \cdot 5t^4 + 5t^5 = 0 \quad \checkmark$,
t^7: $t^2 \cdot 42t^5 - 5t \cdot 7t^6 + 5t^7 = 12t^7 \neq 0 \quad \times$. So t and t^5 are solutions.
c) $W[t, t^5] = \det\begin{pmatrix} t & t^5 \\ 1 & 5t^4 \end{pmatrix} = 5t^5 \neq 0$, so these solutions generate the general solution.

5. a)

$$\begin{vmatrix} 3 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 4 & 1 & 0 & 2 \\ 2 & 2 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 3 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 4 & 1 & 0 & 2 \\ -2 & 2 & 0 & 0 \end{vmatrix} = -\begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & 2 \\ -2 & 2 & 0 \end{vmatrix} = -\begin{vmatrix} 4 & 1 \\ -2 & 2 \end{vmatrix} + 2\begin{vmatrix} 3 & 2 \\ -2 & 2 \end{vmatrix} = 10$$

Do not worry about the remaining problems.