Mathematics 38 **Differential Equations**
Examination 1 **October 4, 2010, 8:30–10:30**

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the blue book in order to receive full credit. Please box your answers and cross out any work you do not want graded. Make sure to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on the exam. *Good luck!*

1. (16 points) Consider the equation

$$(t+1)x' + tx = 0$$

a. Is it separable?
b. Is it linear?
c. Is it homogeneous?
d. Give the largest interval containing 0 where the equation is normal.
e. Write the equation in standard form.
f. Solve the equation.
g. Find a solution with $x(0) = 2$.
h. Why is the solution unique? Because the given differential equation is normal at $t = 0$. **Or:** Because the existence-and-uniqueness theorem applies since $f(t,x) = -\dfrac{t}{t+1}x$ is continuous at $(0,2)$ and so is its x-derivative.

2. (5 points)
a. Determine the largest rectangular region of the t-x plane that contains the point (t_0, a) and on which the hypotheses of the existence and uniqueness theorem hold for the given o.d.e.
b. By solving the o.d.e., find the largest interval of t-values on which the solution of the o.d.e. that satisfies $x(t_0) = \alpha$ is defined and has values in the rectangular region found in (a).

$$\frac{dx}{dt} = \cot t \qquad x\left(\frac{\pi}{2}\right) = 0$$

3. (10 points) Use Cramer's rule to solve for x only.

$$\begin{aligned} x + 2y - 3z &= 1 \\ 2x + 2y + 3z &= 2 \\ x - \ \ y + \ \ z &= 0 \end{aligned}$$

4. (10 points) The Wronskian of t, t^2, t^3 is 0 when $t = 0$ (check this). Why are t, t^2, t^3 independent?

5. (10 points) Solve

$$(D^3 + 3D^2 + 3D + 1)x = 0$$

$$x(0) = x'(0) = x''(0) = 0$$

6. (4 points) Use exponential shift to compute

$$(D^2 - 7D + 5)t^2 e^{2t}$$

7. (5 points) Find the annihilator of

$$e^t + \sin 2t - 3$$

8. (10 points) Solve $(4D^2 + 1)(D^2 + 2D + 2)x = 0$
9. (10 points) Solve $(3D^2 + 2D - 1)x = 2\sin t$
10. (10 points) Solve $(D - 1)x = e^t$
11. (10 points) Use variation of parameters to solve

$$(2D^2 - 4D + 2)x = e^t$$

No credit by any other method.

END OF EXAMINATION