1. (4 points) Determine whether the spring modeled by $(mD^2 + bD + k)x = 0$ with $m = 1$ gram, $k = 5$ dynes/cm and $b = 4$ gram/s is undamped, underdamped, critically damped, or overdamped.

2. (6 points) Determine whether the system

$$\begin{aligned} x' &= -ty - z + t^2 \\ y' &= -\frac{x}{t} - \frac{z}{t} + 1 \\ z' &= x - ty \\ w' &= tx - y\sqrt{3} + z\sin t + w \end{aligned}$$

is linear. If it is linear

a. determine whether it is homogeneous, **b.** determine its order, and **c.** write it in matrix form.

3. (8 points) Given the differential equation

$$(D^2 + 4D + 3)x = 3t + 7 \qquad \text{(N)}$$

a. find the equivalent system $(\mathrm{S_N})$,

b. the general solution of (N) is $x(t) = c_1e^{-t} + c_2e^{-3t} + t + 1$. *You do not need to verify this.* Use the general solution of (N) to obtain each component of the general solution of $(\mathrm{S_N})$,

c. write $(\mathrm{S_N})$ in matrix form,

d. write the general solution of $(\mathrm{S_N})$ in the form $\vec{x} = c_1\vec{h}_1(t) + \cdots + c_n\vec{h}_n(t) + \vec{p}(t)$.

4. (8 points) In parts **a.** and **b.** you are given a matrix A, a vector-valued function $\vec{E}(t)$ and formulas describing a collection of solutions of the nonhomogeneous system $D\vec{x} = A\vec{x} + \vec{E}(t)$. In each case decide whether the collection is complete.

a. $A = \begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix}$, $\quad \vec{E}(t) = \begin{pmatrix} 2e^{-t} \\ -e^{-t} \end{pmatrix}$: $\quad \begin{cases} x_1 = 2c_1e^{-2t} + c_2e^{-t} \\ x_2 = -c_1e^{-2t} - c_2e^{-t} + e^{-t} \end{cases}$.

b. $A = \begin{pmatrix} 5 & -3 & 0 \\ 3 & -5 & 0 \\ 0 & 1 & 2 \end{pmatrix}$, $\quad \vec{E}(t) = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$: $\quad \begin{cases} x_1 = 6c_1e^{4t} - 2c_2e^{-4t} \\ x_2 = 2c_1e^{4t} - 6c_2e^{-4t} \\ x_3 = c_1e^{4t} + c_2e^{-4t} - 2 \end{cases}$.

5. (6 points) Rewrite $f(t) = \begin{cases} 4t + 1 & t < 12 \\ 0 & 12 \le t < 30 \\ t^2 & t \ge 30 \end{cases}$ in unit step function notation.

Examination continues on next page

6. **a.** (5 points) Compute $\mathscr{L}[e^{1-2t}]$ using the definition. *No credit by any other method*

b. (3 points) State for which values of s this Laplace transform is defined.

7. (8 points) Find $\cos 5t * 4$.

8. (8 points) Find the Laplace transform of $f(t) = 5te^{7t} \sin 2t$.

9. (16 points) Find the inverse Laplace transform of

a. $\dfrac{s+2}{s^2+4s+5}$.

b. $\dfrac{5s}{(s^2+25)^2}$.

10. (20 points) Solve using the Laplace transform. *No credit by any other method.*

a. $x'' + 4x' + 4x = t^2 e^{-2t}, \qquad x(0) = x'(0) = 0.$

b. $(D-1)x = \begin{cases} 0 & t < 2 \\ 1 & t \geq 2 \end{cases}, \qquad x(0) = 1.$

11. (8 points) Check this set of vectors for linear independence: $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 2 \\ 1 \\ 4 \\ 3 \\ 2 \end{pmatrix} \quad \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}.$

END OF EXAMINATION