Mathematics 38 **Differential Equations**
Exam II **November 2, 2009**

Tufts UNIVERSITY

1. (15 points) Solve the differential equation

$$\big((t-1)D^2 - tD + 1\big)x = (t-1)e^t$$

by variation of parameters. You may use that the solution of the associated homogeneous differential equation is $H(t) = c_1 t + c_2 e^t$. [No credit for other methods or if you start with the wrong $H(t)$.]

2. (10 points) Find $\mathscr{L}[1]$ *using the definition.*

3. (10 points) Find the following convolutions *using the definition*

a. $e^{2t} * e^{2t}$,

b. $e^{2t} * e^{3t}$.

4. (15 points) Find the Laplace transform of

a. $\dfrac{d^3}{dt^3} t^4 e^{2t}$,

b. $\begin{cases} 0 & t < \pi \\ \cos\frac{t}{2} & t \geq \pi \end{cases}$,

c. $t^2 e^{5t} \sin 3t$.

5. (15 points) Find the inverse Laplace transform of

a. $\dfrac{s+5}{s^2 - 2s + 5}$,

b. $\dfrac{e^{-s}}{s(s+3)}$,

c. $\dfrac{5}{(s^2+5)^2}$.

6. (15 points) Solve the initial-value problem

$$(D^2 + 3D + 2)x = \begin{cases} e^{2t} & t < 1 \\ 0 & t \geq 1 \end{cases}, \qquad x(0) = x'(0) = 0$$

using the Laplace transform. No credit by any other method.

a. Write the system in matrix form.

b. Find the general solution of the differential equation.

7. (10 points) Determine whether

$$\begin{pmatrix} 1 \\ 5 \\ 7 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ -3 \\ -4 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

are linearly independent.

END OF EXAMINATION