Mathematics 38 **Tufts UNIVERSITY** **Differential Equations**
Exam II **March 15, 2010**

1. (8 points) Find $\mathscr{L}[t]$ *using the definition.*

2. (14 points) Find the following convolutions *using the definition.*

a. $e^{3t} * e^{3t}$,

b. $t * e^{5t}$.

3. (6 points) Find $\mathscr{L}^{-1}\left[\dfrac{1}{s(s^2+1)}\right]$ using convolutions.

4. (21 points) Find the inverse Laplace transform of the following functions.

a. $\dfrac{s-3}{s^2-4s+6}$.

b. $\dfrac{s^2+5s-12}{(s-3)^2(s+1)}$.

c. $\dfrac{se^{\pi s}}{s^2+4}$ (you must simplify your answer).

5. (10 points) Solve $(D^2+1)x = \begin{cases} \sin 2t & t < \pi \\ -\sin 2t & t \geq \pi \end{cases}$, $x(0) = x'(0) = 0$.

6. (15 points) Consider the differential equation

(N) $$D(D+1)(D-1)x = t.$$

The corresponding homogeneous equation $D(D+1)(D-1)x = 0$ has the general solution $H(t) = c_1 + c_2e^{-t} + c_3e^{t}$.

a. Find the general solution of (N) by whatever method you prefer.

b. Write (N) as a 3×3 nonhomogeneous system.

c. Use **a.** to find the general solution of the system in **b.**.

7. (21 points) Find the Laplace transform of the following functions.

a. $te^{3t}\cos 7t$.

b. $\begin{cases} 1 & 0 \leq t < 4 \\ 0 & 4 \leq t < 8 \\ e^{2t} & 8 \leq t \end{cases}$.

c. $\cos^2 t$. (Hint: Use a double-angle formula.)

8. (5 points) Compute $\mathscr{L}[\sin^2 t] + \mathscr{L}[\cos^2 t]$.

END OF EXAMINATION