

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the blue book in order to receive full credit. Please box your answers and **cross out any work you do not want graded**. Make sure to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on the exam. *Good luck!*

1. (10 points) Use the definition of the Laplace transform to find

$$\mathcal{L}[t \cdot e^{3t}]$$

No credit by any other method.

2. (15 points) Find the Laplace transform of the following functions:

a.  $e^{-t} \cos 3t$

b.  $te^{2t} \sin 2t$

c. 
$$\begin{cases} 0 & t < 1 \\ t & 1 \leq t < 2 \\ 2(t-1)^2 & 2 \leq t \end{cases}$$

3. (15 points) Find the inverse Laplace transform of the following functions:

a.  $\frac{e^{-4s}}{s+9}$

b.  $\frac{2s-1}{s^2+4s+5}$

c.  $\frac{1}{s^2(s^2-4)}$

4. (10 points) Use the definition of convolution to find

$$t * e^t$$

No credit by any other method

5. (10 points) Compute  $\mathcal{L}^{-1}\left[\frac{1}{s(s+4)}\right]$  using convolution. No credit by any other method.

6. (20 points) Solve:

a.  $(D^2 + 2D + 2)x = \begin{cases} 0 & t < \pi \\ e^{-t} \sin t & \pi \leq t \end{cases} \quad x(0) = x'(0) = 0$

b.  $(D^2 + 4D + 4)x = te^{-2t} \quad x(0) = 0, \quad x'(0) = 1$

7. (10 points) Check for independence

a.  $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

b.  $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 5 \\ 6 \\ 7 \\ 8 \end{pmatrix}, \begin{pmatrix} 7 \\ 6 \\ 5 \\ 4 \end{pmatrix}$

8. (10 points)

a. Find the eigenvalues of the matrix  $\begin{pmatrix} 1 & -1 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 2 \end{pmatrix}$

- b. Find the corresponding eigenvector(s) for each eigenvalue.