

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the blue book in order to receive full credit. Please box your answers and **cross out any work you do not want graded**. Make sure to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on the exam. *Good luck!*

1. (15 points) The  $3 \times 3$  matrix

$$A = \begin{pmatrix} 3 & -3 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

has a (triple) eigenvalue of 1 (you do not have to verify this). Find the general solution of

$$D\vec{x} = A\vec{x}$$

2. (10 points) Find all solutions of

$$\begin{pmatrix} 1 & 2 & 1 & -1 & -1 \\ 2 & 2 & 2 & -3 & -2 \\ -1 & 0 & -1 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

or explain why the system has no solutions.

3. (10 points) Solve  $D\vec{x} = A\vec{x}$ , where

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

4. (15 points) Solve

$$D\vec{x} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \vec{x}$$

5. (15 points) Solve

$$D\vec{x} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \vec{x} + \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad \vec{x}(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

6. (20 points) For the system

$$\frac{dx}{dt} = 4x - 2x^2 - xy$$

$$\frac{dy}{dt} = -y + xy$$

- Find the equilibria (and determine their stability)
- Classify each equilibrium as an attractor, a repeller, or neither of these.
- Draw the phase portrait of the linearization at each equilibrium.
- Draw the phase portrait of the entire system.

7. (15 points) For the system

$$\frac{dx}{dt} = x^2y + y^3$$

$$\frac{dy}{dt} = x^3 + xy^2$$

- Verify that  $G(x, y) = y^2 - x^2$  is a constant of motion.
- Find the equilibria of the system.
- Find the critical points of  $G$  and classify them as extremum or saddle.
- Classify each equilibrium of the system as stable or unstable.