Mathematics 38 **Differential Equations**
Exam III **November 19, 2007**

Tufts UNIVERSITY

No calculators, notes, or books are allowed. Please make sure all electronic devices you carry are turned off and put away out of sight.

Remember to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on this exam. Good luck!

1. (10 points) Find all solutions of

$$\begin{aligned} 2x+\ 1y-\ \ z+\ \ w-\ \ u &= 2 \\ 2x+\ 2y-3z+2w-2u &= 1 \\ -y+2z-\ \ w+\ \ u &= 0 \end{aligned}$$

2. (10 points) Check whether $\begin{pmatrix}1\\2\\3\\4\end{pmatrix}$, $\begin{pmatrix}0\\1\\2\\3\end{pmatrix}$, $\begin{pmatrix}1\\3\\5\\6\end{pmatrix}$ are linearly independent.

3. (10 points)

a. Find as many linearly independent eigenvectors as possible of $A = \begin{pmatrix}1&0&4\\0&3&0\\1&0&1\end{pmatrix}$.

[You may use and you do not need to verify that the characteristic polynomial of A is $-(\lambda-3)^2(\lambda+1)$.]

b. Find the general solution of $D\vec{x} = A\vec{x}$.

4. (20 points) Find the general solution of $D\vec{x} = \begin{pmatrix}2&0&-2\\0&1&0\\2&0&2\end{pmatrix}\vec{x}$.

5. (20 points) Find the general solution of $D\vec{x} = A\vec{x}$, where $A = \begin{pmatrix}-1&1&4\\-2&2&4\\-1&0&4\end{pmatrix}\vec{x}$.

[You may use and you do not need to verify that the characteristic polynomial of A is $(2-\lambda)^2(1-\lambda)$.]

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6. (10 points) Solve

$$\begin{aligned} x_1' &= -x_1 + x_2 \\ x_2' &= -6x_1 + 4x_2 \\ x_3' &= x_2 - x_3 \end{aligned} \quad , \qquad x_1(0) = 1, \quad x_2(0) = 2, \quad x_3(0) = 3.$$

[You may use and you do not need to verify that $A = \begin{pmatrix} -1 & 1 & 0 \\ -6 & 4 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ *has eigenvalue* -1 *with eigenvector* $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$*, eigenvalue* 1 *with eigenvector* $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ *and eigenvalue* 2 *with eigenvector* $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$*.]*

7. (10 points) Solve $D\vec{x} = A\vec{x}$, where $A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 4 \end{pmatrix}$.

[You may use and you do not need to verify that the characteristic polynomial of A *is* $(2-\lambda)^3(4-\lambda)$*, and that* $\begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ *is an eigenvector for the eigenvalue* 4*.]*

8. (10 points) Find the general solution of

$$\begin{aligned} x' &= -2x - y + e^{-3t} \\ y' &= 2x - y + z \\ z' &= 2y - 2z - 2e^{-3t} \end{aligned}$$

[You may use and you do not need to verify that the characteristic polynomial of $\begin{pmatrix} -2 & -1 & 0 \\ 2 & -1 & 1 \\ 0 & 2 & -2 \end{pmatrix}$ *is* $-(\lambda+2)^2(\lambda+1)$*, that* $\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ *is an eigenvector for the eigenvalue* -1 *and that* $\begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$ *and* $\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$ *are generalized eigenvectors for the eigenvalue* -2*.]*

END OF EXAMINATION