



Mathematics 38
Exam III

Differential Equations
April 9, 2007

No calculators, notes, or books are allowed. Please make sure all electronic devices you carry are turned off and put away out of sight.

Remember to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on this exam. Good luck!

1. (10 points) Solve

$$1w + 3x - 15y + 7z = 1$$

$$1w + 4x - 19y + 10z = 3$$

$$2w + 5x - 26y + 11z = 0$$

2. (10 points) Check whether $\begin{pmatrix} 1 \\ 2 \\ 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 3 \\ 4 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 8 \\ 7 \\ 5 \end{pmatrix}$ are linearly independent.

3. (5 points) Find the characteristic polynomial of $\begin{pmatrix} 3 & 0 & 0 \\ -4 & 6 & 2 \\ 16 & -15 & -5 \end{pmatrix}$.

4. (10 points) The characteristic polynomial of $A = \begin{pmatrix} 4 & -2 & 1 \\ 2 & 0 & 1 \\ 2 & -2 & 3 \end{pmatrix}$ is $(\lambda - 2)^2(\lambda - 3)$. (*You do not need to verify this!*) Find as many linearly independent eigenvectors as possible. Find the general solution of $D\vec{x} = A\vec{x}$ and explain why this is the general solution.

5. (20 points) Find the general solution of $D\vec{x} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} \vec{x}$.

6. (20 points) Find the general solution of $D\vec{x} = \begin{pmatrix} 2 & -1 & -4 \\ 0 & 2 & -4 \\ 0 & 1 & -2 \end{pmatrix} \vec{x}$.

7. (15 points) Consider the differential equation

$$(E) \quad (D^3 - D)x = 0$$

a. Change (E) into a 3×3 system $D\vec{x} = A\vec{x}$ with a 3×3 matrix A .

b. Find the characteristic polynomial of A .

c. $\lambda = 0$ is an eigenvalue of A . (*You do not need to verify this.*) Find an eigenvector of A corresponding to $\lambda = 0$.

8. (10 points) Solve $D\vec{x} = \begin{pmatrix} -3 & 1 \\ -2 & 0 \end{pmatrix} \vec{x}$, $\vec{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

END OF EXAMINATION