Math 38 | Tufts University
Department of Mathematics
Exam 2 | April 13, 2009

No calculators, notes, books, pagers, mobile phones or other electronic devices are allowed on the exam. All answers should be in terms of real numbers and functions. You must **show all your work** to receive credit. *You are required to sign your exam book. With your signature, you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

1. (10 points) Solve: $\begin{pmatrix} 1 & 2 & 1 & -1 & -1 \\ 2 & 2 & 2 & -3 & -2 \\ -1 & 0 & -1 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$

2. (20 points) Consider the matrix $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 4 \end{pmatrix}$

   You may use the following facts (*you do not need to verify them*)

   (a) The characteristic polynomial $-(1-\lambda)(2-\lambda)(1+\lambda)(4-\lambda)$

   (b) The eigenvector corresponding to $\lambda = 4$ is $\begin{pmatrix} 1 \\ 15 \\ 3 \\ 15 \end{pmatrix}$

   Solve the initial value system $D\mathbf{x} = A\mathbf{x} \quad \mathbf{x}(0) = \begin{pmatrix} 2 \\ 16 \\ 5 \\ 15 \end{pmatrix}$

3. (10 points) Solve $D\mathbf{x} = \begin{pmatrix} 5 & -4 \\ 10 & -7 \end{pmatrix} \mathbf{x} \qquad \mathbf{x}(0) = \begin{pmatrix} 70 \\ 40 \end{pmatrix}$

Exam continues on other side

4. (15 points) If

$$A = \begin{pmatrix} -1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

Solve $D\mathbf{x} = A\mathbf{x}$

5. (15 points) If

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{pmatrix}$$

The general solution of $D\mathbf{x} = A\mathbf{x}$ is

$$h = c_1 \ e^t \begin{pmatrix} 1 \\ t \\ t \end{pmatrix} + c_2 \ e^t \begin{pmatrix} 0 \\ 1-t \\ -t \end{pmatrix} + c_3 \ e^t \begin{pmatrix} 0 \\ t \\ 1+t \end{pmatrix}$$

You do not have to verify this.

Solve $D\mathbf{x} = A\mathbf{x} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

6. (20 points) Sketch the following phase portraits

(a) $\dfrac{dx}{dt} = 1 - x^2$

(b) $D\mathbf{x} = \begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix} \mathbf{x}$

7. (10 points) Check for independence

$$\begin{pmatrix} 1 \\ 0 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 3 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ -15 \\ -11 \\ 12 \end{pmatrix}$$

End of Exam