Mathematics 38 **Differential Equations**
Exam III **April 12, 2010**

Tufts UNIVERSITY

No calculators, notes, or books are allowed. Please make sure all electronic devices are turned off and out of sight.
Show all work and cross out work you do not want graded!
Remember to sign your blue book.
With your signature you are pledging that you have neither given nor received assistance on this exam. Good luck!

Please put the answers to problems 1–2 on the blue book cover in the corresponding box, as shown here:

1	Yes
2	w.
3	e.
4	c.
5	a.n.
6	cos(t)
7	high
8	yes
9	
10	
T	

1. (10 points, no partial credit) Consider the system $D\vec{x} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{pmatrix} \vec{x}$. The general solution is

a. $c_1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_3 e^{-2t} \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$

b. $c_1 e^{t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_3 e^{-2t} \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$

c. $c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + c_3 e^{-2t} \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$

d. $c_1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} + c_3 e^{-2t} \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$

e. None of the above

2. (10 points, no partial credit) Are the following 3 vectors linearly independent? $\begin{pmatrix} 1 \\ 0 \\ -2 \\ 3 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} -3 \\ -1 \\ -4 \\ 4 \end{pmatrix}$

(*Answer "yes" or "no".* **Don't guess! A wrong answer will receive –5 points!**)

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For the remaining problems show all work; an answer without an explanation, even if correct, will receive no credit.
If you solve any part of a problem by inspection, you must make clear why the solution works.

3. (20 points) The general solution of $D\vec{x} = \begin{pmatrix} 5 & -3 & 0 \\ 3 & -5 & 0 \\ 0 & 1 & 2 \end{pmatrix} \vec{x} + \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$ is

$$\vec{x} = c_1 e^{4t} \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix} + c_2 e^{-4t} \begin{pmatrix} -2 \\ -6 \\ 1 \end{pmatrix} + c_3 e^{2t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}.$$

You do not have to verify this. Find the solution that satisfies $\vec{x}(0) = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$.

4. (20 points) Solve $D\vec{x} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \vec{x}$.

5. (20 points) Solve $D\vec{x} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \vec{x}$.

6. (20 points) Solve $D\vec{x} = \begin{pmatrix} 2 & -1 & -4 \\ 0 & 2 & -4 \\ 0 & 1 & -2 \end{pmatrix} \vec{x}$.

END OF EXAMINATION