

No calculators, notes, or books are allowed. Please make sure all electronic devices are turned off and out of sight. Show all work and cross out work you do not want graded!

Remember to sign your blue book.

With your signature you are pledging that you have neither given nor received assistance on this exam. Good luck!

Please write the answers to problems 1–8 on the cover of the blue book in the corresponding box, as shown below.

1. (3 points, answer only, no partial credit) Can the system below be solved by Cramer's rule?

$$x_1 + x_2 + x_3 = 0$$

$$2x_1 + 3x_2 + x_3 = 0$$

$$-x_1 - 3x_2 + x_3 = 0$$

2. (3 points, answer only, no partial credit) Consider the following system of equations:

$$x_1 + x_2 + x_3 = 0$$

$$2x_1 + 3x_2 + x_3 = 0$$

$$-x_1 - 3x_2 + x_3 = 0$$

Choose one answer: The system has

- a. a unique solution, b. no solution, c. more than 1 solution, d. None of the above.

3. (3 points, answer only, no partial credit) Choose one answer.

$$\det \begin{pmatrix} 5 & 1 & \sin t & t^2 + 3 & 1 \\ 0 & 4 & e^t & e^t & 0 \\ 0 & 0 & 3 & \ln t & 8 \\ 0 & 0 & 0 & 2 & \sqrt{t} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} =$$

- a. 5, b. $2\sqrt{t}$, c. 120, d. $120 - \sin t - \frac{1}{4}e^t - \frac{1}{4}(t^2 + 3)e^t$, e. None of the above.

4. (3 points, answer only, no partial credit) Choose one answer.

The functions e^t , e^{-t} , e^{t+1} are

- a. linearly dependent b. linearly independent, c. None of the above.

5. (3 points, answer only, no partial credit) Consider the differential equation

$$t^4 x'' - 2xx' = 0.$$

Choose every function from the following that is a solution of this differential equation.

- a. 0, b. 1, c. t , d. t^2 , e. t^3 , f. t^4 , g. None of the above.

6. (3 points, answer only, no partial credit) Suppose $g(t)$ is a continuous function. Solve the initial-value problem $x' = g(t)x$, $x(1) = 0$.

(Hint: Think before applying standard techniques.)

7. (3 points, answer only, no partial credit) For the differential equation

$$t \frac{dx}{dt} - x = t^3$$

state the order of the differential equation, whether the differential equation is linear, homogeneous or nonhomogeneous, and whether it has constant coefficients or not. If the equation is linear, find the largest interval containing 1 on which the differential equation is normal.

8. (3 points, answer only, no partial credit) For the initial-value problem

$$(t - 1)x' + x = 0, \quad x(1) = 0$$

decide whether the Existence-and-Uniqueness Theorem applies.

9. (6 points)

a. Find the general solution of $(D^2 + 4)(D^2 - 4)(D - 4)^2 x = 0$.

b. Find a simplified guess for a particular solution of $(D - 3)^2 x = e^{3t}$.

10. (7 points) Find the general solution of $D\vec{x} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \vec{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

You may use that the general solution of the associated homogeneous system is $c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} e^{2t} \\ e^{2t} \end{pmatrix}$.

1	Yes
2	n.
3	e.
4	c.
5	a. n.
6	$\cos(t)$
7	high
8	Yes
9	
10	
T	

11. (8 points)

a. Find the Laplace transform of $f(t) = \begin{cases} 0 & t < \pi \\ t \sin(2t) & t \geq \pi \end{cases}$.

b. Find the inverse Laplace transform of $F(s) = \frac{9}{s^3 + 6s^2 + 9s}$

12. (10 points) Use the Laplace transform to solve $(D^2 + 2D + 2)^2 x = 0$ with $x(0) = x'(0) = x''(0) = 0$ and $x'''(0) = 2$.
No credit will be given for a solution using any other method.

13. (10 points) Find the general solution of the system

$$\begin{aligned}x_1' &= x_1 - x_2 \\x_2' &= -x_2 + 3x_4 \\x_3' &= x_3 \\x_4' &= -x_1 + x_2\end{aligned}$$

Without verifying it, you may use that

$$\begin{aligned}x_1 &= 3c_1 - c_2 e^{2t} - c_3 e^{-2t} \\x_2 &= 3c_1 + c_2 e^{2t} - 3c_3 e^{-2t} \\x_3 &= 0 \\x_4 &= c_1 + c_2 e^{2t} + 3c_3 e^{-2t}\end{aligned}$$

is a solution for any choice of c_1, c_2, c_3 .

14. (20 points) Given the system

$$(S) \quad \begin{aligned}\frac{dx}{dt} &= x - x^2 - xy \\ \frac{dy}{dt} &= 2xy - y\end{aligned}$$

- Find the equilibrium points.
- Find the linearization matrix $A_{(x,y)}$ of (S).
[Check your work carefully! You will not get credit for the following parts if $A_{(x,y)}$ is wrong.]
- Classify each equilibrium as an attractor, a repeller or neither of these.
- Classify each equilibrium as stable or unstable.
- Draw the phase portrait.

15. (15 points) Consider the system

$$\begin{aligned}\frac{dx}{dt} &= x^3 + y \\ \frac{dy}{dt} &= y^3 - x\end{aligned}$$

- Find all equilibria.
- Decide whether $E(x, y) = -x^2 - y^2$ is a constant of motion for this system.
- Decide whether $E(x, y) = -x^2 - y^2$ is a Lyapunov function for this system.
- Classify each equilibrium as an attractor, a repeller, or neither of these.
- Determine the stability of each equilibrium.
- Draw the phase portrait of the *linearization* of each equilibrium.
- For each equilibrium determine whether the Hartman–Grobman Theorem applies.
- Decide whether this system of differential equations has a closed integral curve.