

No calculators, notes, or books are allowed. Please make sure all electronic devices are turned off and out of sight.

Show all work and cross out work you do not want graded!

Remember to sign your blue book.

With your signature you are pledging that you have neither given nor received assistance on this exam. Good luck!

Part I: No partial credit

Please write the answers to problems 1–6 on the cover of the blue book to the right of the corresponding box, as shown below.

1. (4 points) Are the vectors $\begin{pmatrix} 1 \\ 0 \\ -1 \\ 5 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 1 \\ 2 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 5 \\ -7 \\ 2 \\ 1 \end{pmatrix}$ linearly independent?

2. (4 points) Are the vectors $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$ linearly independent?

3. (4 points) Are the vectors $\begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 7 \\ -2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}$ linearly independent?

4. Consider the differential equation $t \frac{dx}{dt} = x - 2$.

- a. (2 points) Is this equation linear?
- b. (2 points) What is the largest interval containing $t = 1$ on which the equation is normal?
- c. (2 points) Does the existence and uniqueness theorem apply for $x(0) = 2$?
- d. (2 points) How many solutions of this differential equation satisfy $x(0) = 2$?
(None? Exactly one? Infinitely many?)
- e. (2 points) How many solutions of the equation satisfy $x(0) = 0$?

5. (5 points) Find the general solution of $(D^2 + 2)^2 D(D^2 - 1)x = 0$.

6. (6 points) Find the annihilator of

- a. t^4 ,
- b. $te^{2t} + \sin t$.

7. (11 points) The general solution of $((t - 1)D^2 - tD + 1)x = 0$ for $t > 1$ is $h(t) = c_1 t + c_2 e^t$. You do not have to verify this.

Find the solution of $((t - 1)D^2 - tD + 1)x = (t - 1)^2 e^t$ that satisfies $x(2) = 1$, $x'(2) = e^2$.

1	Yes
2	v.
3	e.
4	c.
5	a.n.
6	$\cos(t)$
7	high
8	Yes
9	
10	
T	

8. (12 points)

a. Find the eigenvalues of $A = \begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & 1 \\ -1 & -2 & 4 \end{pmatrix}$.

b. Find the corresponding eigenvectors. (An answer of $\vec{0}$ in **b.** or **c.** will forfeit all credit.)

c. Find the general solution of $D\vec{x} = A\vec{x}$.

9. (11 points) The eigenvalue of $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{pmatrix}$ is 1 (triple). Find the general solution of $D\vec{x} = A\vec{x}$.

10. (11 points)

a. Write $f(t) = |(t-1)(t-2)|$ as an expression in step-function notation.

b. Compute the Laplace transform of $f(t)$. (Remember to give reasons!)

11. (10 points) Compute $\mathcal{L}^{-1} \left[\frac{s}{(s^2+1)(s^2+4)} \right]$.

12. Consider the system

$$\begin{aligned} \frac{dx}{dt} &= 2y - x - 3 \\ \frac{dy}{dt} &= 4 - 2x - y. \end{aligned}$$

a. (2 points) Find the equilibria of the system.

b. (2 points) Decide whether for this system $E(x, y) = x^2 - 2x + y^2 - 4y$ is a constant of motion, a Lyapunov function or neither of these.

c. (2 points) Classify all equilibria as attractors, repellers or neither of these.

d. (3 points) Show that the system has no closed integral curves.

e. (3 points) Draw the phase portrait.