Midterm Exam 2

Instructions. No calculators, notes, or books are allowed. Please make sure all electronic devices are turned off and out of sight. Show all work and cross out work you do not want graded! *Remember to sign your blue book.* With your signature you are pledging that you have neither given nor received assistance on this exam. **Good luck!**

1. (15 points): Use the method of undetermined coefficients to find a particular solution of the equation

$$x'' + x' - 2x = -18te^{-2t}.$$

2. (10 points): Solve the following equation by the method of variation of parameters

$$2x'' + 2x = \sec t.$$

No credit by any other method.

3. (10 points): Find the Laplace transform of the following functions:

(a) $te^{-t}\sin 2t$

(b) $\begin{cases} 2, & \text{if } t < 2, \\ t, & \text{if } 2 \leq t < 3, \\ e^{t-3} + 3, & \text{if } 3 \leq t. \end{cases}$

4. (10 points): Find the inverse Laplace transform of the following functions:

(a) $\dfrac{e^{-s}(1+s)}{2(s^2+1)}$

(b) $\dfrac{s}{s^2 - 2s + 2}$

5. (10 points): Use the following steps to compute the convolution $t * (\sin t - e^t)$:

(a) find the Laplace transform of $t * (\sin t - e^t)$;

(b) find a partial fraction decomposition of the Laplace transform obtained in Part (a);

(c) find the inverse Laplace transform of the partial fraction decomposition found in Part (b).

6. (10 points): Compute $\mathcal{L}^{-1}\left[\frac{1}{s^2(s-2)}\right]$ using the convolution formula. No credit by any other method.

7. (15 points): Solve the following initial value problem

$$x'' - 4x' + 5x = f(t),$$
$$x(0) = 0,\ x' = 1,$$

where $f(t) = \begin{cases} 5, & \text{if } t < 1 \\ 0, & \text{if } 1 \le t. \end{cases}$

8. (10 points): Reduce the following higher order linear differential equations to equivalent systems (DO NOT SOLVE):

(a) $x'' + (e^t + t)x' = \sec t$

(b) $(D+1)^3 x = t$

9. (10 points): Determine whether the vector functions

$$h_1(t) = \begin{pmatrix} 3e^{4t} \\ e^{4t} \end{pmatrix} \quad \text{and} \quad h_2(t) = \begin{pmatrix} 3e^{-4t} \\ e^{-4t} \end{pmatrix}$$

generate the general solution of the system

$$x_1' = 5x_1 - 3x_2,$$
$$x_2' = 3x_1 - 5x_2.$$

Justify your answer.