

## Midterm Exam 3

**Instructions.** No calculators, notes, or books are allowed. Please make sure all electronic devices are turned off and out of sight. Show all work and cross out work you do not want graded! *Remember to sign your blue book.* With your signature you are pledging that you have neither given nor received assistance on this exam.

**Good luck!**

1. (10 points): Check following sets of vectors for independence (NO PARTIAL CREDIT):

(a)  $\begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$

(b)  $\begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 2 \\ 4 \\ 8 \end{pmatrix}, \begin{pmatrix} 6 \\ 8 \\ 8 \\ 10 \end{pmatrix}$

2. (15 points): (a) Find the eigenvalues of the matrix  $A = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & 1 & 3 \end{pmatrix}$

(b) Find eigenvectors for each eigenvalue of the matrix  $A$ .

3. (10 points): The matrix  $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$  has eigenvectors  $v = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$  and  $w = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ .

Find the corresponding eigenvalues.

4. (10 points): Use the row reduction method to solve the following systems:

$$\begin{aligned}x_1 + 2x_2 + x_3 - x_4 - x_5 &= 2 \\2x_1 + 2x_2 + 2x_3 - 3x_4 - 2x_5 &= 1 \\-x_1 & \quad -x_3 + 2x_4 + x_5 = 1.\end{aligned}$$

No credit for any other method.

5. (20 points): Solve the system of linear differential equations  $D\vec{x} = A\vec{x}$ , where

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{pmatrix}$$

6. (15 points): The matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

has a triple eigenvalue  $\lambda = 1$  (You do not have to verify this). Find the generalized eigenvectors of  $A$ .

7. (20 points): Solve the following initial value problem:

$$D\vec{x} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \vec{x} \quad \vec{x}(0) = \begin{pmatrix} 10 \\ 20 \\ 10 \end{pmatrix}$$