

Review Problems

2. a. $x_1 = 5c_1e^{3t} + c_2e^{-t} + 18e^t$, $x_2 = c_1e^{3t} + c_2e^{-t} + 10e^t$

b. $x_1 = -5e^{3t} - 2e^{-t} + 18e^t$, $x_2 = -e^{3t} - 2e^{-t} + 10e^t$

4. a. $x_1 = c_1e^{2t} - c_2e^t + (t-1)e^{2t} + e^{3t}$, $x_2 = c_2e^t + e^{2t}$

b. $x_1 = -e^t + (t+3)e^{2t} + e^{3t}$, $x_2 = e^t + e^{2t}$

6. a. $x_1 = c_1(1+t)e^{2t} - c_2te^{2t} + c_3te^{2t}$

$$x_2 = c_1(2t+t^2)e^{2t} + c_2(1-t-t^2)e^{2t} + c_3(t+t^2)e^{2t}$$

$$x_3 = c_1(t+t^2)e^{2t} - c_2t^2e^{2t} + c_3(1+t^2)e^{2t}$$

b. $x_1 = (2-2t)e^{2t}$

$$x_2 = (8-t^2)e^{2t}$$

$$x_3 = (4+2t-t^2)e^{2t}$$

8. a. $x_1 = c_2(\cos 2t + \sin 2t) + c_3(\sin 2t - \cos 2t) - 1$

$$x_2 = c_1e^{2t} + 2c_2\cos 2t + 2c_3\sin 2t + te^{2t} - 2$$

$$x_3 = c_1e^{2t} + te^{2t}$$

b. $x_1 = 4 \cos 2t - 2 \sin 2t - 1$

$$x_2 = 2 \cos 2t - 6 \sin 2t + (t+1)e^{2t} - 2, \quad x_3 = (t+1)e^{2t}$$

10. a. $x_1 = c_1 e^t + c_3 t e^t$

$$x_2 = c_1 \frac{t^2}{2} c_2 e^t + c_3 \frac{t^3}{6} e^t + c_4 t e^t$$

$$x_3 = c_3 e^t$$

$$x_4 = c_1 t e^t + c_3 \frac{t^2}{2} e^t + c_4 e^t$$

b. $x_1 = t e^t$

$$x_2 = \left(\frac{t^3}{6} + t \right) e^t$$

$$x_3 = e^t$$

$$x_4 = \left(\frac{t^2}{2} + 1 \right) e^t$$

12. a. $x_1 = c_1 \sin 3t - c_2 \cos 3t + c_3(\sin 3t - 3t \cos 3t)$

$$+ c_4(-\cos 3t - 3t \sin 3t)$$

$$x_2 = c_1 \cos 3t + c_2 \sin 3t + 3c_3 t \sin 3t - 3c_4 t \cos 3t$$

$$x_3 = 2c_3 \sin 3t - 2c_4 \cos 3t$$

$$x_4 = 2c_3 \cos 3t + 2c_4 \sin 3t$$

b. $x_1 = 5 \sin 3t - 6t \cos 3t + 3t \sin 3t$

$$x_2 = 3 \cos 3t + \sin 3t + 6t \sin 3t + 3t \cos 3t$$

$$x_3 = 4 \sin 3t + 2 \cos 3t$$

$$x_4 = 4 \cos 3t - 2 \sin 3t$$

14. a. $x = c_1 - c_2 e^{-2t} - c_3 e^{-8t} + 2t - L - \frac{13}{4}$

$$v = 2c_2 e^{-2t} + 8c_3 e^{-8t} + 2$$

(continued on next page)

$$y = c_1 + 2t$$

b. $x = c_1 + c_2(1 + 2t)e^{-2t} + c_3te^{-2t} + 2t - L - 10$

$$v = -4tc_2e^{-2t} + c_3(1 - 2t)e^{-2t} + 2$$

$$y = c_1 + 2t$$

c. $x = c_1 - c_2 \sin 2t - c_3 \cos 2t + 2t - L - 8$

$$v = 2c_2 \cos 2t + 2c_3 \sin 2t + 2$$

$$y = c_1 + 2t$$