

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. You must show all your work in the blue book in order to receive full credit. Please circle your answers and cross out any work you do not want graded. Make sure to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on the exam.

1. (10 points) When interest is compounded continuously, the rate of change of the principal $x(t)$ is proportional to the principal. The constant of proportionality is called the interest rate.
 - (a) Set up a differential equation to model the principal $x(t)$ with an interest rate of 5% a year.
 - (b) If the initial deposit is \$1,000, how much is $x(20)$? Use $e \approx 2.78$

2. (10 points) Solve the initial value problem

$$x^2 \frac{dx}{dt} = t^2, \quad x(1) = 2 .$$

3. (10 points) Solve the initial value problem

$$\frac{dx}{dt} - tx = t, \quad x(0) = \frac{1}{2} .$$

4. (16 points)

- (a) Check that for $t > 0$ or for $t < 0$ the equation

$$tx' - 2x = 0$$

has solutions $x = ct^2$ (c an arbitrary constant).

- (b) Show that for any constants c_1 and c_2

$$x(t) = \begin{cases} c_1 t^2 & t \geq 0 \\ c_2 t^2 & t < 0 \end{cases}$$

is a solution of $tx' - 2x = 0$ on $(-\infty, \infty)$.

- (c) Why does this not contradict the existence and uniqueness theorem?

- (d) Show that any solution to $tx' - 2x = 0$ must satisfy $x(0) = 0$.

Exam continues on other side

5. (10 points) Draw the phase portrait of

$$\frac{dx}{dt} = (x^2 - 9)(1 - x)^2$$

and label the equilibria as attractors, repellers or neither.

6. (14 points) The functions $h_1 = t^2$, $h_2 = t^{-1}$ and $h_3 = 1$ are solutions of

$$(*) \quad (t^2 D^3 + 2tD^2 - 2D)x = 0.$$

YOU DO NOT HAVE TO VERIFY THIS.

- (a) Find the general solution of (*) and explain why this is the general solution.
(b) Solve the non-homogenous equation

$$(t^2 D^3 + 2tD^2 - 2D)x = t^2.$$

7. (10 points) Solve $(D - 1)^2(D + 3)x = 0$

$$x(0) = 0, \quad x'(0) = 1, \quad x''(0) = 0.$$

8. (20 points) Solve by any method

(a) $(9D^2 - 1)x = t.$

(b) $(D^2 + 1)x = \sin t.$

End of Exam